Tulsa Math Teachers' Circle

Latin, Magic, and Sudoku Squares

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1. Magic Matrix, Martin Gardner, "*Aha! Gotcha*", Paradoxes to Puzzle and Delight

I. Copy this 4x4 matrix on your paper and number the cells from 1-16

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

- Draw a circle around any number you like.
- Draw a vertical line through the column containing your number.
- Draw a horizontal line through the row containing your number
- Choose any number not already crossed out and circle it.
- Draw lines through the number's column and row as before.
- Choose a third number in the same way and cross out its row and column.
- Circle the single number that remains.
- Now add the four numbers you selected (the circled ones).
- Are you ready? Your four selected numbers add to 34!! Correct?
- Was I able to influence your selection? How did I know?

If this were an addition table, what would my number generators be? Sum them. What do you notice?

(1 2 3 4 across the top to signify each column, and 0 4 8 12 down side to signify each row)

Could I make another size matrix? Could its numbers look random?

Try to find the generators for this additive matrix.

5	2	6	3	1	4	(Try 4 1 5 2 0 3 above the columns and 1 5 2 4 0 3 down
9	6	10	7	5	8	the side by the rows)
6	3	7	4	2	5	What number will you get when selecting 6 numbers as
8	5	9	6	4	7	you did before? (30) Can the sum be anything?
4	1	5	2	0	3	Could negative numbers be used? Rational, irrational?
7	4	8	5	3	6	

How about complex numbers? Could you multiply instead of add? To see more information on magic matrices, consult the second chapter of Martin Gardner's "*Scientific American Book of Mathematical Puzzles and Diversions*"

Magic Squares

Magic squares have been around since the Ming Dynasty in China, for over 3,000 years. Traditional magic squares are nxn squares formed from the digits 1, 2, …, n^2, such that each row, each column, and each main diagonal sum to the same number, the magic sum.

Legend: After a great flood, in order to calm the Lo River, the Chinese people made an offering to the river god. Each time they made an offering, a turtle would appear from the water. A little boy noticed symbols on the back of the turtle that represented the numbers from 1-9. They were arranged like this:

4	9	2
3	5	7
8	1	6

Describe all the number patterns and relationships that you can find in the Lo Shu square.

Questions:

1. What is the sum of the numbers on the turtle?
2. What is the sum of any nxn magic square? $[(1 + n^2)\, n^2]/2$

   What is the magic sum of any 3x3 magic square? (Since there are n rows with the same magic sum, divide the total sum by n, giving $[(1 + n^2)\, n^2]/2n =$ **$[(1 + n^2)n]/2$ (Check this answer when n=3 to see if you are correct)**

   How is the magic sum related to the middle number?
3. What is the sum of the four corners of the magic square? How is this sum related to the middle number?

4. Can you prove that the sum and magic sum relationships in 2 and 3 above hold for any 3x3 magic square? Use the magic square with entries a-i below for your proof.

a	b	c
d	e	f
g	h	i

Hint: Sum the three following equations using M to be the magic sum for the matrix.

$M = a + e + i$

$M = d + e + f$

$M = g + e + c$

5. Now look at these two non-traditional rational number magic squares. What are their respective magic sums? Can you complete the squares with fractions in simplest form? Use what you learned above.

___	___	5/6
___	1/2	___
1/6	___	___

___	___	___
___	1/4	___
___	___	___

6. What is the magic sum for a 5x5 magic square? Show how you got the answer.