Tulsa Math Teachers' Circle
Counting Problems
October 6, 2016

1. In how many different ways can you form a subset of the set {1, 2, 3, 4, 5, 6, 7, 8, 9} which contains at least one prime number? (3 Points)

2. How many subsets does a set with 8 elements have, including the empty set and the whole set itself? (1 Point)

3. How many squares in the plane have two or more vertices I the set S = {(0,0), (0,1), (0,2), (1,0), (1,1), (2,0)}? (3 Points)

4. How many three-digit numbers can be built from the digits in the list 2, 3, 5, 5, 5, 6, 6? (6 Points)

5. A decimal integer is called odd-looking if all of its digits are odd. How many odd-looking 5-digit numbers are there? (1 Point)

6. A palindrome on the alphabet {H, T} is a sequence of H's and T's which reads the same from left to right as from right to left. Thus HTH, HTTH, HTHTH, and HTHHTH are palindromes of lengths 3, 4, 5, and 6 respectively. Let P(n) denote the number of palindromes of length n over {H, T}. For how many values of n is $1{,}000 < P(n) < 10{,}000$? (State Math Contest) (3 Points)

7. Marcus has a bag of green marbles, a bag of red marbles, and a bag of blue marbles. In how many different patterns can he place two of these marbles in a row? Three marbles? Four marbles? (1 Point)

8. There are 20 Americans, 10 Australians, and 10 Austrians. Fourteen are needed for a math team. How many ways can a math team be made if:
   a) Nationality is irrelevant?
   b) The team must be all American?
   c) The team must have 5 Americans, 5 Australians, and 4 Austrians?
   d) Nationality is irrelevant, but Shaun Cater, one of the Australians, must be on the team? ☺
   (3 Points)

9. How many ways are there to distribute 10 doggie biscuits among 7 dogs? The biscuits are indistinguishable, but the dogs are distinguishable. (6 Points)

10. There are 3 rooms in a dormitory: a single, a double, and a quad. How many ways are there to assign 7 people to the rooms? (1 Point)

11. A permutation $\{a_1, a_2, a_3, a_4, a_5\}$ of $\{1, 2, 3, 4, 5\}$ is heavy-tailed if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations? (2008 AMC 12 B, Problem 21) (3 Points)

12. How many ways are there to arrange 5 red, 5 green, and 5 blue balls in a row so that no two blue balls lie next to each other? (6 Points)

13. There are only 6 letters in the Martian alphabet. All Martian words are exactly 4 letters long.

a. Suppose that any sequence of 4 letters is a valid Martian word. How many words are there in the Martian language?
b. Suppose that any sequence of 4 non-repeating letters is a valid Martian word. How many words are there in the Martian language?
c. Suppose that any sequence of 4 letters that has at least one repetition is a valid Martian word. How many words are there in the Martian Language? (There's an easy way to do this!)

(1 Point)

14. How many ways are there to choose a team of 3 students from a group of 30? (1 Point)

15. How many positive integers less than 500 can be written as the sum of two positive perfect cubes? (3 Points)

16. How many three-digit numbers have the sum of their digits equal to 9 when the 1^{st} digit is not zero? (1 Point)

17. If the coefficients A and B of the equation of a straight line $Ax + By = 0$ are two distinct digits from the numbers 0, 1, 2, 3, 6, 7, then how many distinct straight lines are there? (3 Points)

18. In each of the cases below, we ask how many ways there are to place the chess pieces on a standard chessboard so they cannot attack each other:

a) Two bishops (Bishops attack on the diagonals of a chessboard)
b) Two knights (Knights attack two squares horizontally and one square vertically, or two squares vertically and one square horizontally)
c) Two queens (Queens attack on either rows, columns, or diagonals of a chessboard)
(6 Points)

19. How many diagonals are in a convex n-gon? (1 Point)

20. Suppose that a, b, c in the equation of a straight line ax +by + c = 0 are three distinct elements of the set {-3, -2, -1, 0, 1, 2, 3} and the inclination of the straight line is an acute angle. Then how many distinct lines are there?
(3 Points)

21. Given *n* a positive integer, a plus or minus sign is assigned randomly to each of the integers 1, 2, …, *n*. Let P(*n*) be the probability that the sum of the *n* signed numbers is positive. What is the value of [P(1) + P(2) +…+ P(6)]?
(6 Points)

22. How many positive integers less than 1,000 have an odd number of positive integer divisors? (1 Point)