Lunes, Moons, & Balloons

$+=*/-(=+^)/\in\blacktriangleleft\pm\forall\cong\prod\pi\neq\geq\approx\sum\nabla\leq\lambda*-\exists\mu\sim\infty$

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Warm Up

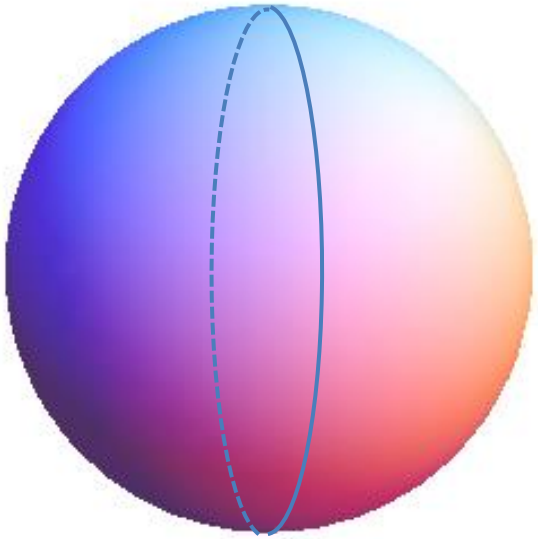
$+=*/-(=+^)/\in\blacktriangleleft\pm\forall\cong\prod\pi\neq\geq\approx\sum\nabla\leq\lambda*-\exists\mu\sim\infty$

- **Circles**
  - **Circumference**
  - **Area**
- **Spheres**
  - **Surface area**
  - **Volume**

Great Circles

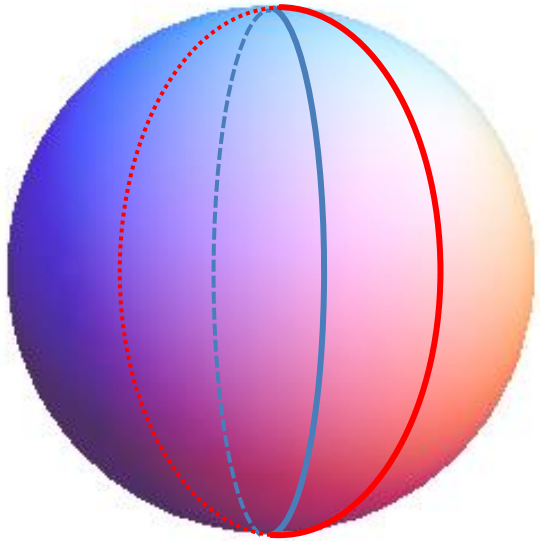
$+=*/-(=+^)/\in\leftarrow\pm\forall\cong\Pi\pi\neq\geq\approx\Sigma\nabla\leq\lambda*-\exists\mu\sim\infty$

- **A great circle is a circle on a spherical surface such that the plane containing the circle passes through the center of the sphere**
  - **Divides the sphere into two congruent hemispheres**



Lunes

- **A lune is created when two great circles intersect.**



Lunes

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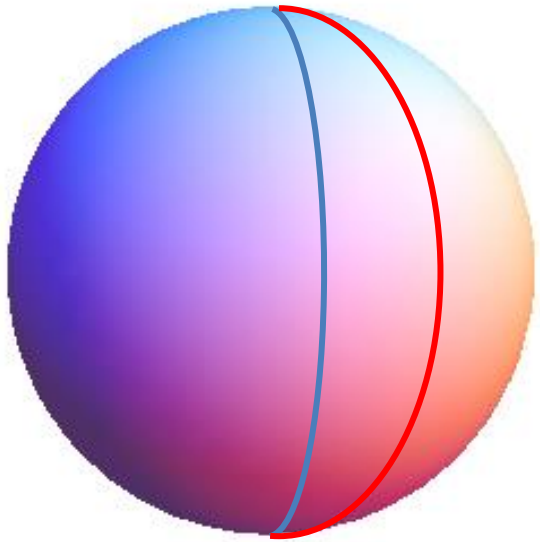

Spherical Area

$+=*/-(=+^)/\in\leftarrow\pm\forall\cong\prod\pi\neq\geq\approx\sum\nabla\leq\lambda*-\exists\mu\sim\infty$

- **The area of a sphere of radius r is**

Spherical Area

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- **The area of a sphere of radius r is $4\pi r^2$.**

Spherical Area

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- **The area of a sphere of radius r is $4\pi r^2$.**
  - **Divide the sphere with a great circle.**
    - **Each congruent hemisphere has an area of**

Spherical Area

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- **The area of a sphere of radius r is $4\pi r^2$.**
  - **Divide the sphere with a great circle.**
    - **Each congruent hemisphere has an area of $2\pi r^2$.**
  - **Divide the sphere with another great circle, which meets the first at right angles.**
    - **Each congruent lune has an area of**

Spherical Area

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- **The area of a sphere of radius r is $4\pi r^2$.**
  - **Divide the sphere with a great circle.**
    - **Each congruent hemisphere has an area of $2\pi r^2$.**
  - **Divide the sphere with another great circle, which meets the first at right angles.**
    - **Each congruent lune has an area of πr^2.**
  - **Divide each of the lunes into two by bisecting the angle.**
    - **Each congruent lune has an area of**

Spherical Area

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  - **Divide the sphere with a great circle.**
    - **Each congruent hemisphere has an area of $2\pi r^2$.**
  - **Divide the sphere with another great circle, which meets the first at right angles.**
    - **Each congruent lune has an area of πr^2.**
  - **Divide each of the lunes into two by bisecting the angle.**
    - **Each congruent lune has an area of $\frac{\pi r^2}{2}$.**

Area of a Lune

$+=*/-(=+^)/\in\leftarrow\pm\forall\cong\prod\pi\neq\geq\approx\sum\nabla\leq\lambda*-\exists\mu\sim\infty$

- **Let's divide a hemisphere into *q* equal lunes.**
  - **What is the lunar angle of each lune?**
  - **What is the area of each lune?**
  - **Take the union of *p* of these lunes.**
    - **What is the lunar angle of the union?**
    - **What is the area of the union?**
  - **What is the relationship between the lunar angle and the area?**

Spherical Triangles

- **Assume the model is a sphere of radius 1 ft.**
  - **Choose a particular great circle (Equator) and mark off an arc *AB* of length $\frac{\pi}{2}$.**
  - **At each endpoint construct a perpendicular (geodesic) segment and extend the two segments until they meet.**
    - **Why must they meet? Where will they meet? Call this point *C.***
      - **What is the sum of the angles of ΔABC?**
      - **Is ΔABC equilateral?**

Spherical Triangles

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- **Continuing with the model of the sphere …**
  - **At point *C*, form an angle of $\frac{\pi}{3}$ rads (60°) with *AC* as one side.**
  - **Extend the other side until it meets *AB*. Call that point *D*.**
    - **What is the sum of the angles of ΔADC?**
    - **What is the distance from *A* to *D*?**
    - **What is the distance from *C* to *D*?**
  - **Let *M* be the midpoint of *AB*. Can you construct a triangle with base *AM* that is similar to ΔABC? Can you construct any other triangle that is similar but not congruent to ΔABC?**

Questions

$+=*/-(=+^)/\in\leftarrow\pm\forall\cong\prod\pi\neq\geq\approx\sum\nabla\leq\lambda*-\exists\mu\sim\infty$

- **Are there parallel great circles?**
- **Can you find a formula that relates the area of a spherical triangle to the sum of its angles?**