Lunes, Moons, & Balloons

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Warm Up

- Circles
  - Circumference
  - Area

- Spheres
  - Surface area
  - Volume
Great Circles

A great circle is a circle on a spherical surface such that the plane containing the circle passes through the center of the sphere.

– Divides the sphere into two congruent hemispheres
Lunes

A lune is created when two great circles intersect.
Lunes

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Spherical Area

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Spherical Area

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Spherical Area

• The area of a sphere of radius $r$ is $4\pi r^2$.
  – Divide the sphere with a great circle.
    • Each congruent hemisphere has an area of
Spherical Area

- The area of a sphere of radius $r$ is $4\pi r^2$.
  - Divide the sphere with a great circle.
    - Each congruent hemisphere has an area of $2\pi r^2$.
  - Divide the sphere with another great circle, which meets the first at right angles.
    - Each congruent lune has an area of...
Spherical Area

• The area of a sphere of radius $r$ is $4\pi r^2$.
  – Divide the sphere with a great circle.
    • Each congruent hemisphere has an area of $2\pi r^2$.
  – Divide the sphere with another great circle, which meets the first at right angles.
    • Each congruent lune has an area of $\pi r^2$.
  – Divide each of the lunes into two by bisecting the angle.
    • Each congruent lune has an area of
Spherical Area

• The area of a sphere of radius \( r \) is \( 4\pi r^2 \).
  – Divide the sphere with a great circle.
    • Each congruent hemisphere has an area of \( 2\pi r^2 \).
  – Divide the sphere with another great circle, which meets the first at right angles.
    • Each congruent lune has an area of \( \pi r^2 \).
  – Divide each of the lunes into two by bisecting the angle.
    • Each congruent lune has an area of \( \frac{\pi r^2}{2} \).
Area of a Lune

• Let’s divide a hemisphere into $q$ equal lunes.
  – What is the lunar angle of each lune?
  – What is the area of each lune?

  – Take the union of $p$ of these lunes.
    • What is the lunar angle of the union?
    • What is the area of the union?

  – What is the relationship between the lunar angle and the area?
Spherical Triangles

- Assume the model is a sphere of radius 1 ft.
  - Choose a particular great circle (Equator) and mark off an arc $AB$ of length $\frac{\pi}{2}$.
  - At each endpoint construct a perpendicular (geodesic) segment and extend the two segments until they meet.
    - Why must they meet? Where will they meet? Call this point $C$.
      - What is the sum of the angles of $\triangle ABC$?
      - Is $\triangle ABC$ equilateral?
Spherical Triangles

• Continuing with the model of the sphere ... 
  – At point $C$, form an angle of $\frac{\pi}{3}$ rads (60°) with $AC$ as one side.
  – Extend the other side until it meets $AB$. Call that point $D$.
    • What is the sum of the angles of $\triangle ADC$?
    • What is the distance from $A$ to $D$?
    • What is the distance from $C$ to $D$?
  – Let $M$ be the midpoint of $AB$. Can you construct a triangle with base $AM$ that is similar to $\triangle ABC$? Can you construct any other triangle that is similar but not congruent to $\triangle ABC$?
Questions

• Are there parallel great circles?
• Can you find a formula that relates the area of a spherical triangle to the sum of its angles?