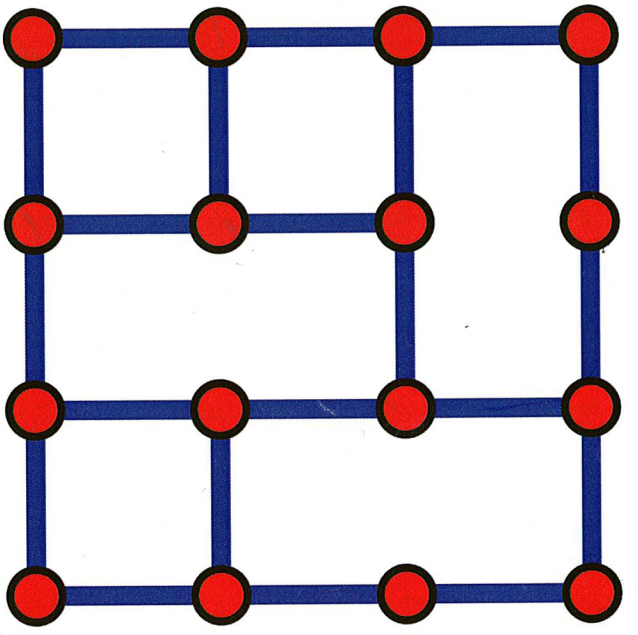
Problem Circle: Possible Paths

by Joshua Zucker

A common math contest problem is to count the number of paths from start to finish in some grid, perhaps one missing a few segments here or there to make it a bit harder to calculate. In this problem, we'll turn that usual formula around!



We'll take square grids and go from the top left corner to the bottom right corner, moving only down or right at each step. Using this rule, in the grid shown, our path has to be of length 6.

If no segments were missing, you could count the number of paths this way: Each path would involve 6 moves, of which 3 would have to be to the right. So, there are $\binom{6}{3} = 20$ possible paths. By deleting different segments, you can get any number of paths smaller than 20 that you would like. In the example at right, there are 7 paths.

Now let's turn our problem around: instead of starting with a grid and asking how many paths there are, we'll tell you how many paths there are and ask you to construct the grid!

1) Find a grid that has exactly 1,000 paths from top left to bottom right.
2) Find a grid with only one missing segment that has exactly 1,000 paths.

For each of these, the goal is to make a "small" grid. That can be measured in a few ways: smallest area, smallest perimeter, or fewest unit paths. Which is hardest? How well can you optimize your grid? Have fun!

WIN A FREE MUG!

Got an answer to this issue's Problem Circle? Send it in and you could win! MTCircular will choose one winner at random from each of the six types of possible answers (smallest area, smallest perimeter, and fewest unit paths, for both problems 1 and 2). We will also send five mugs to the MTC with the most total entries. Submit your answers to problemcircle@aimath.org by August 31 for consideration.