Math Teachers' Circles

and

The Game of Set

Math Teachers' Circle of Oklahoma
October 3, 2013
Judith Covington
judith.covington@lsus.edu
Louisiana State University Shreveport

What is a Math Teacher’s Circle?

The mission of the national Math Teachers’ Circle (MTC) program, developed at the American Institute of Mathematics (AIM), is to establish the foundation for a culture of problem solving among middle school math teachers in the U.S.

For more information visit:
www.mathteacherscircle.org

By fostering the confidence to tackle open-ended math problems, middle school teachers become better equipped to initiate more student-centered, inquiry-based pedagogies in their classrooms. The two primary goals of the program are 1) to engage middle school math teachers in mathematical problem solving and involve them in an ongoing dialogue about math with students, colleagues, and professional mathematicians; and 2) to provide guidance, materials, and resources to middle school math teachers that will enable them to promote open-ended problem solving as a way of learning, thinking about, and practicing mathematics in their classrooms.

Participant comments

- I have enjoyed participating in the North Louisiana Math Teachers' Circle. I find the program to be of immense value in enriching my understanding of mathematics, so that I can then better instruct my students.

- I was able to figure out all of the problems, but I most enjoyed working the abstract problems that were beyond skills of my usual curriculum or where pattern recognition was important. I used some of the problems with my students, but others would be a little beyond their reach. My students most enjoyed the silliness of #4.

4

The Wicked Witch has the power to cast 7 different evil spells: Poison Apple; Endless Sleep; Thorn of Death; Eternal Hiccups; Infinite Ugliness; Agony of Fire; and Warlock’s Plague. She decides to cast 3 spells on poor Snow White.

- **How many different sets of spells could she choose?**
- **If the Magic Mirror chooses the spells at random, what is the probability that Infinite Ugliness will be one of them, and Thorn of Death won’t be one?**

The kids were interested to know about where I got the problems and what we did for the evening. They were amazed that I would voluntarily spend an evening doing math, but it allowed for a class discussion about how solving problems makes me feel like a detective on a case. It's sometimes hard to get kids at this age to understand the beauty of mathematics, but hearing about how I figured out some of the problems and some of my missteps seemed to help them understand how important it is to persevere. If nothing else, they understood that I believe that learning continues throughout life.

- I enjoy our meetings because it keeps my problem-solving skills sharp and I always pick up something I can take to the classroom, whether it's a tricky problem or an activity or a teaching method. I always learn something at each meeting

- I don’t often get the chance to visit and collaborate with other teachers outside of my parish. Through NLMTC, I have been able to enjoy a unique fellowship of middle school math teachers from all of northwest Louisiana. It has been a great experience hearing the points of view and thoughts of others. I have found myself challenged intellectually at times, but I know that we have to stretch to grow. The summer workshop was wonderful, pushing me to learn ways to make some difficult math personally meaningful and relevant for my students. I will continue to be a part of NLMTC as long as we keep meeting.

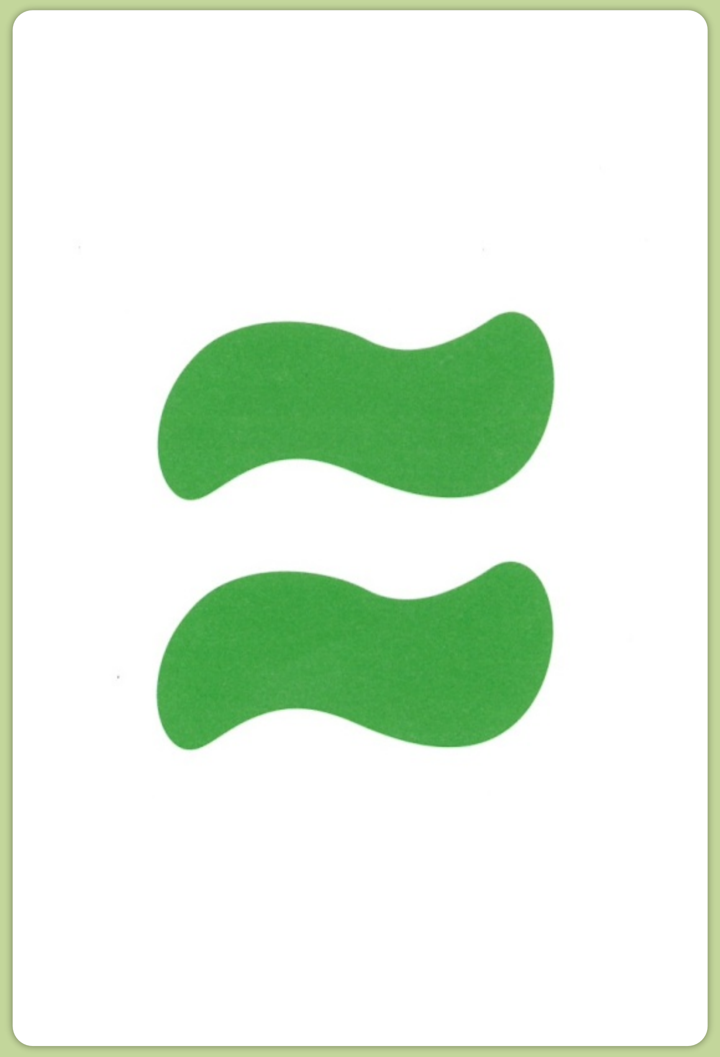

Decimals
Hundreds Board
Welcome!


CAMPUS KAPPAS
543

Enough talking!

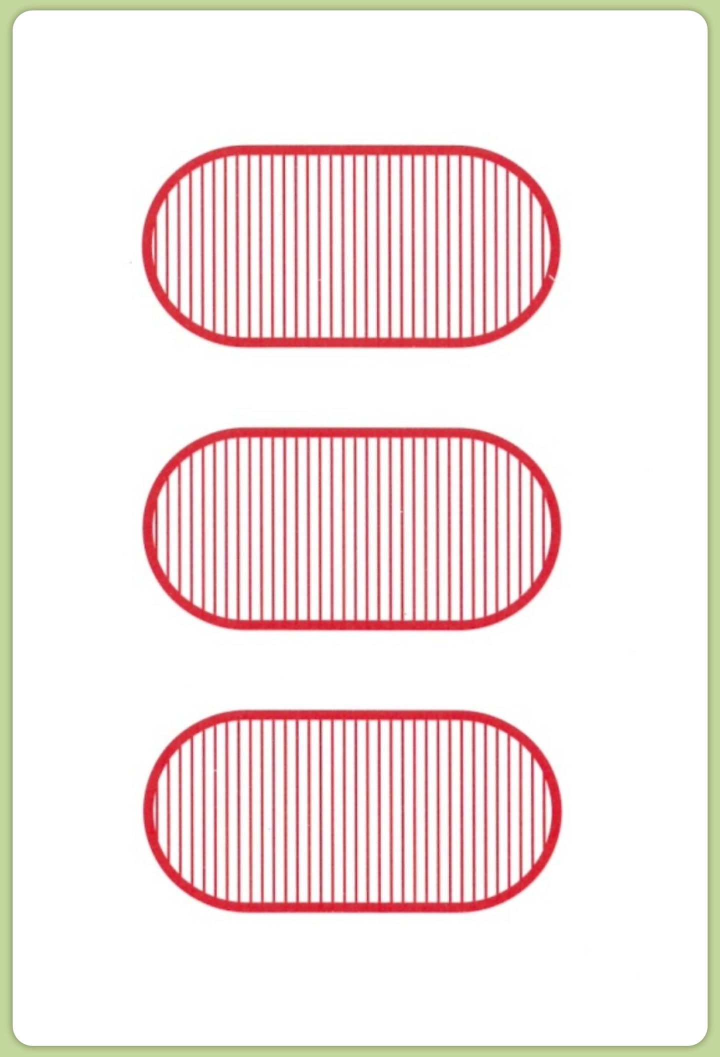
Let's play Set!

What do the cards in a SET game look like?

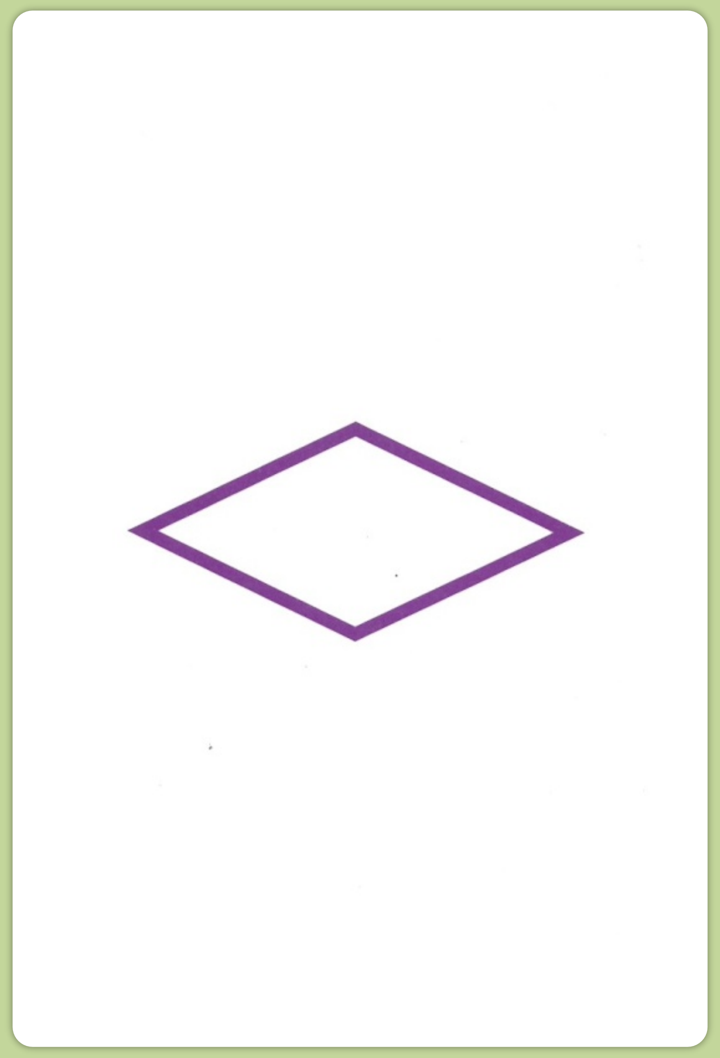


Two solid green squiggles

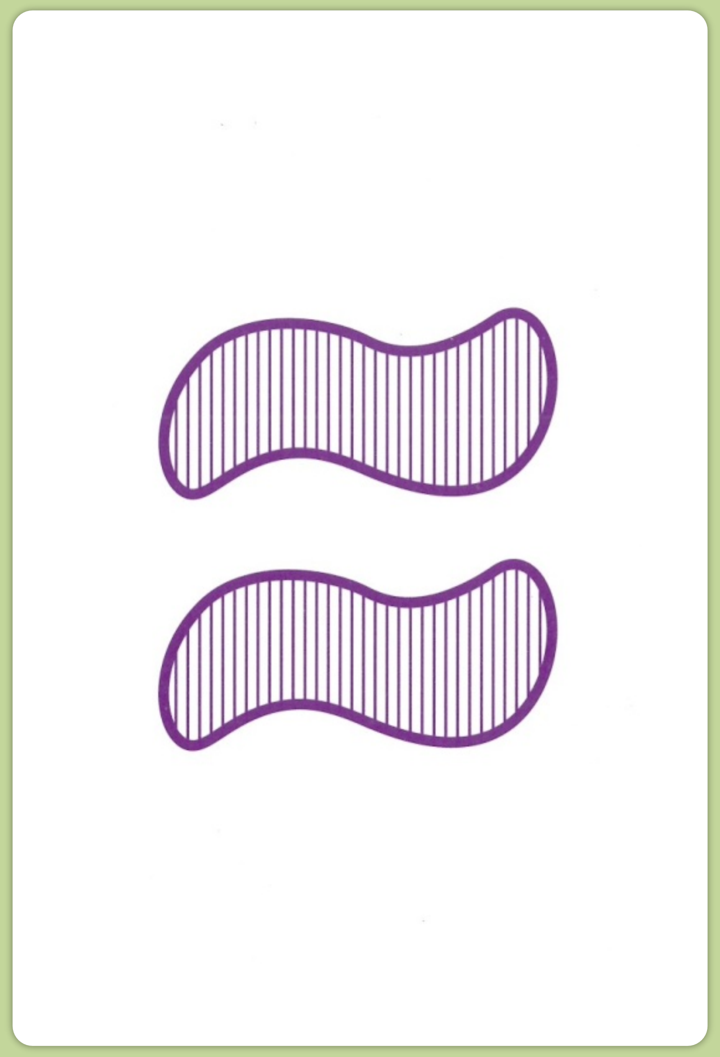
Number, Shading, Color, Shape



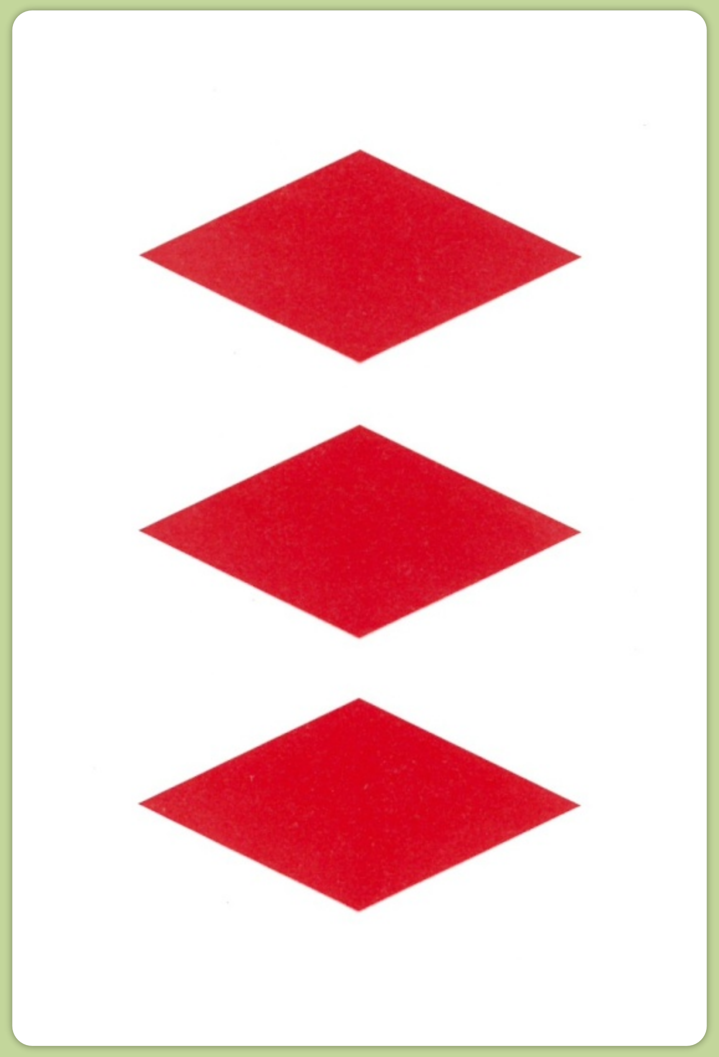
Three striped red ovals



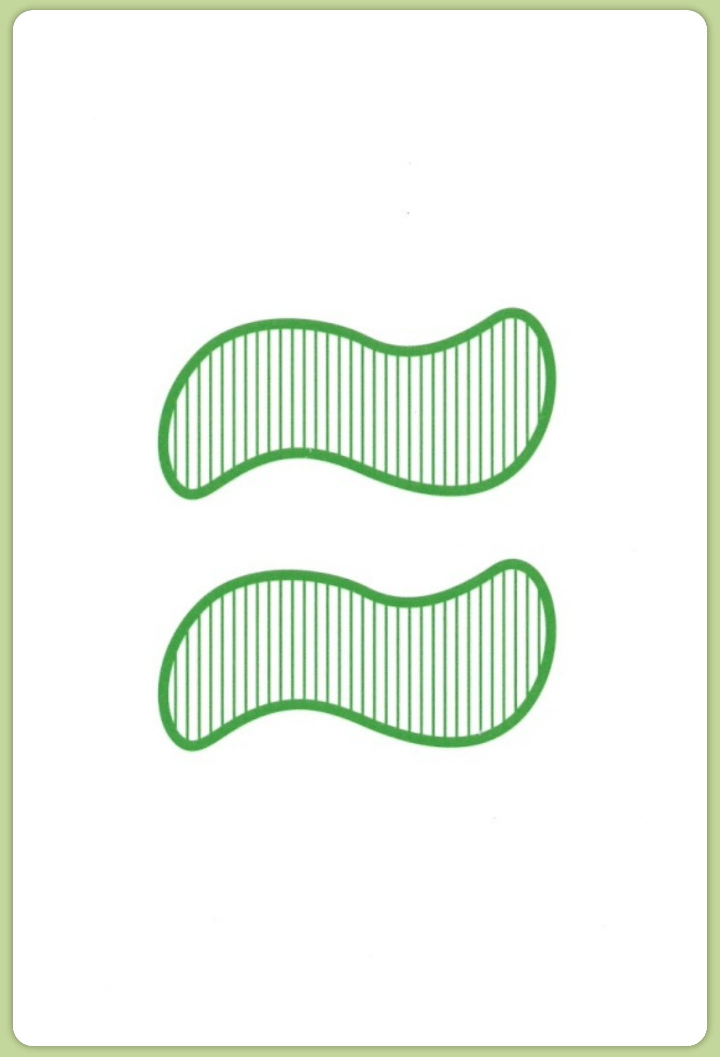
One plain purple diamond



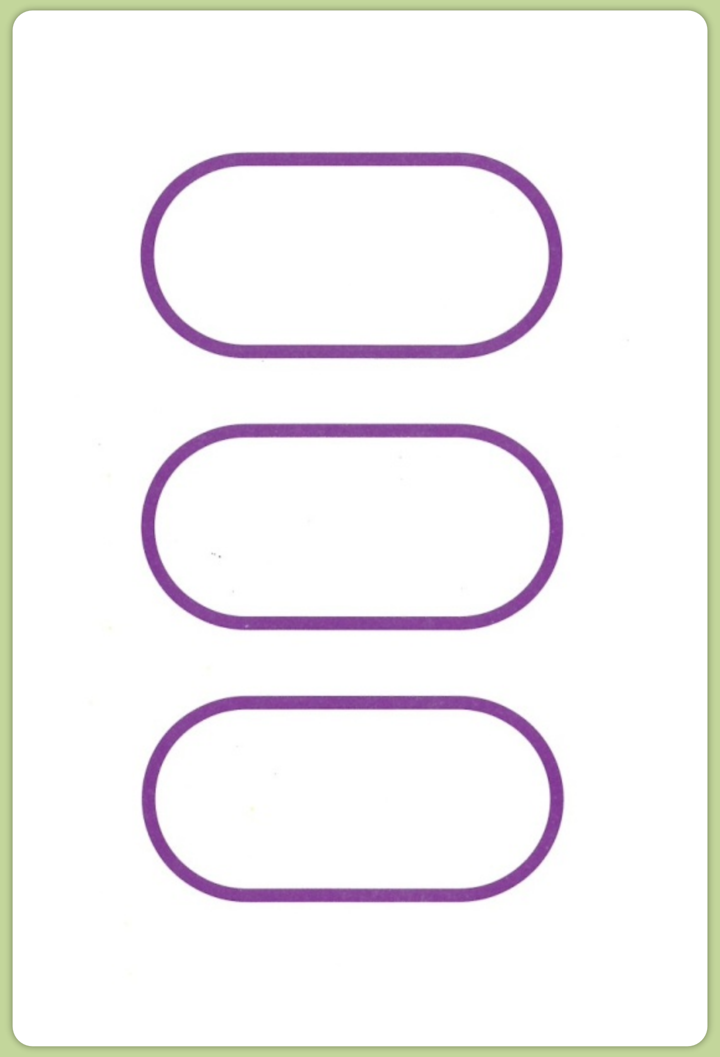
Two striped purple squiggles



Three solid red diamonds



Two striped green squiggles



Three plain purple ovals

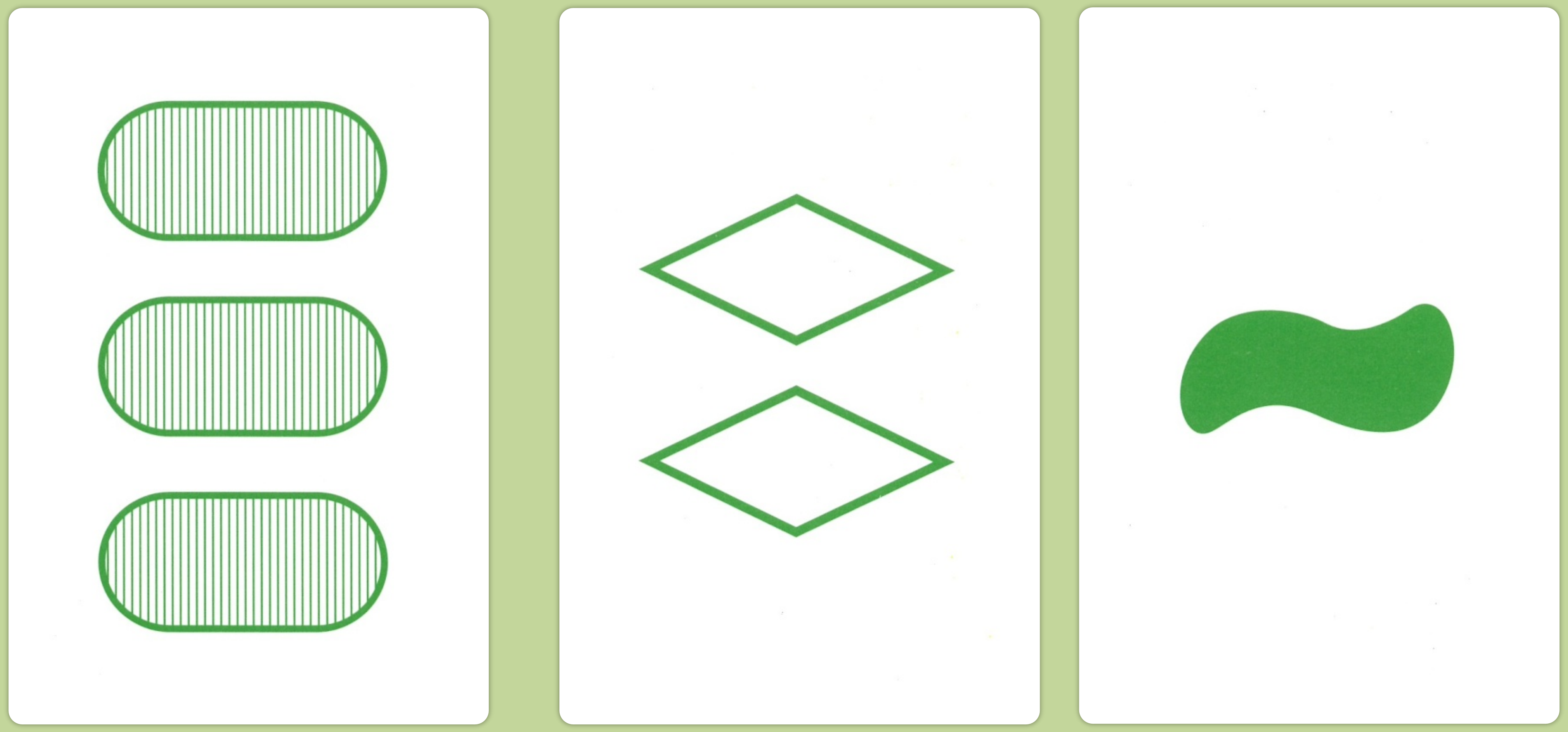
How many cards are in a SET deck?

There are four characteristics on a Set card, number, shading, color and shape. For each characteristic there are three choices, so there are 3x3x3x3 or 81 cards in a Set deck.

A Set is a collection of three cards such that for each characteristic, either all the cards in the trio share the same value for that characteristic, or all of the cards have different values for that characteristic.

For example, if the characteristic is SHAPE, then the three cards should all have the same shape or they should all have different shapes. The same is true for the other three characteristics. So, to determine if a trio is a SET one performs four checks, one for each of the four characteristics. If the trio passes all four of these tests then it is a SET.

Is the following a Set?



Number: All different

Color: All the same

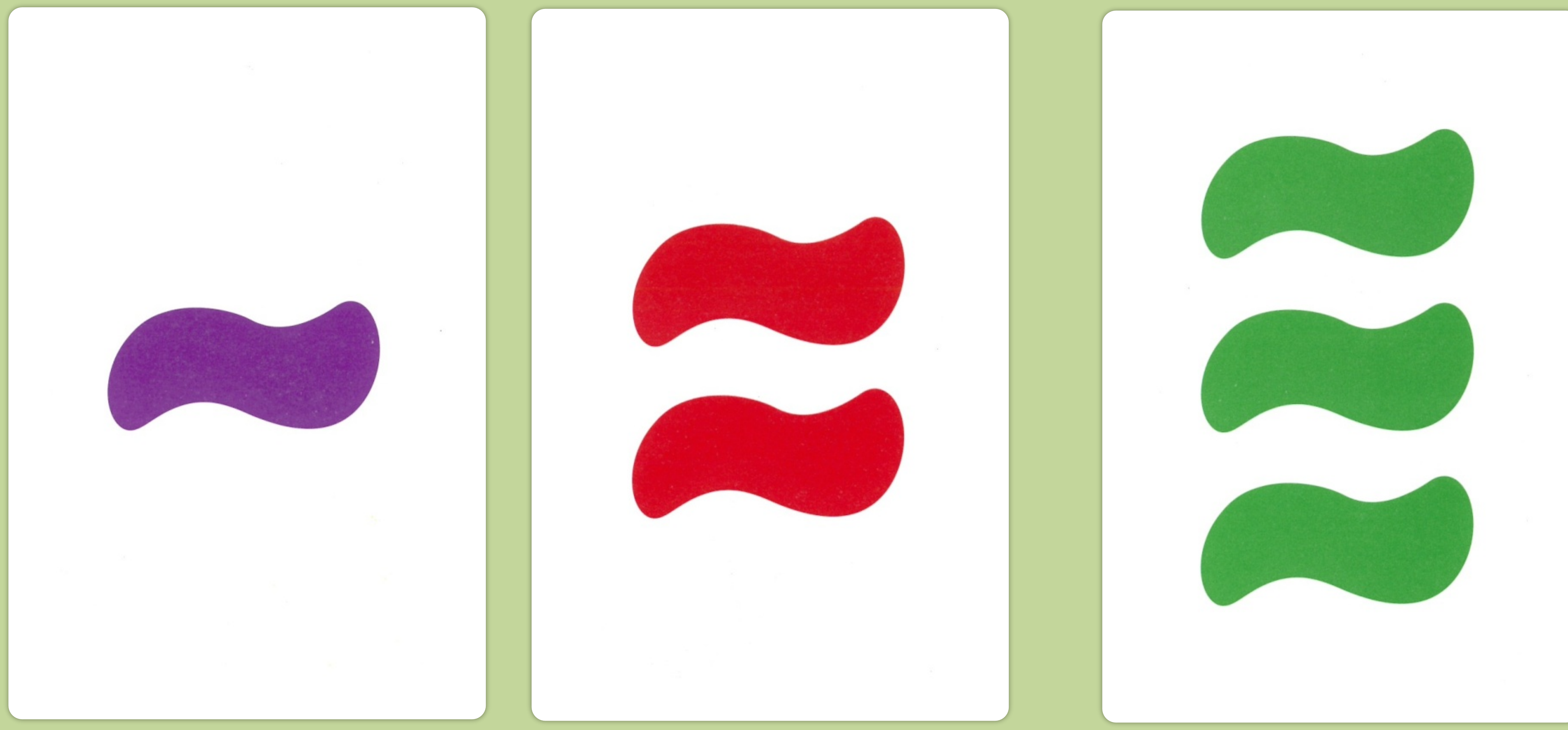
Shading: All different

Shape: All different

Yes!

I call this a “one alike Set.”

Is the following a Set?



Number: All different

Color: All different

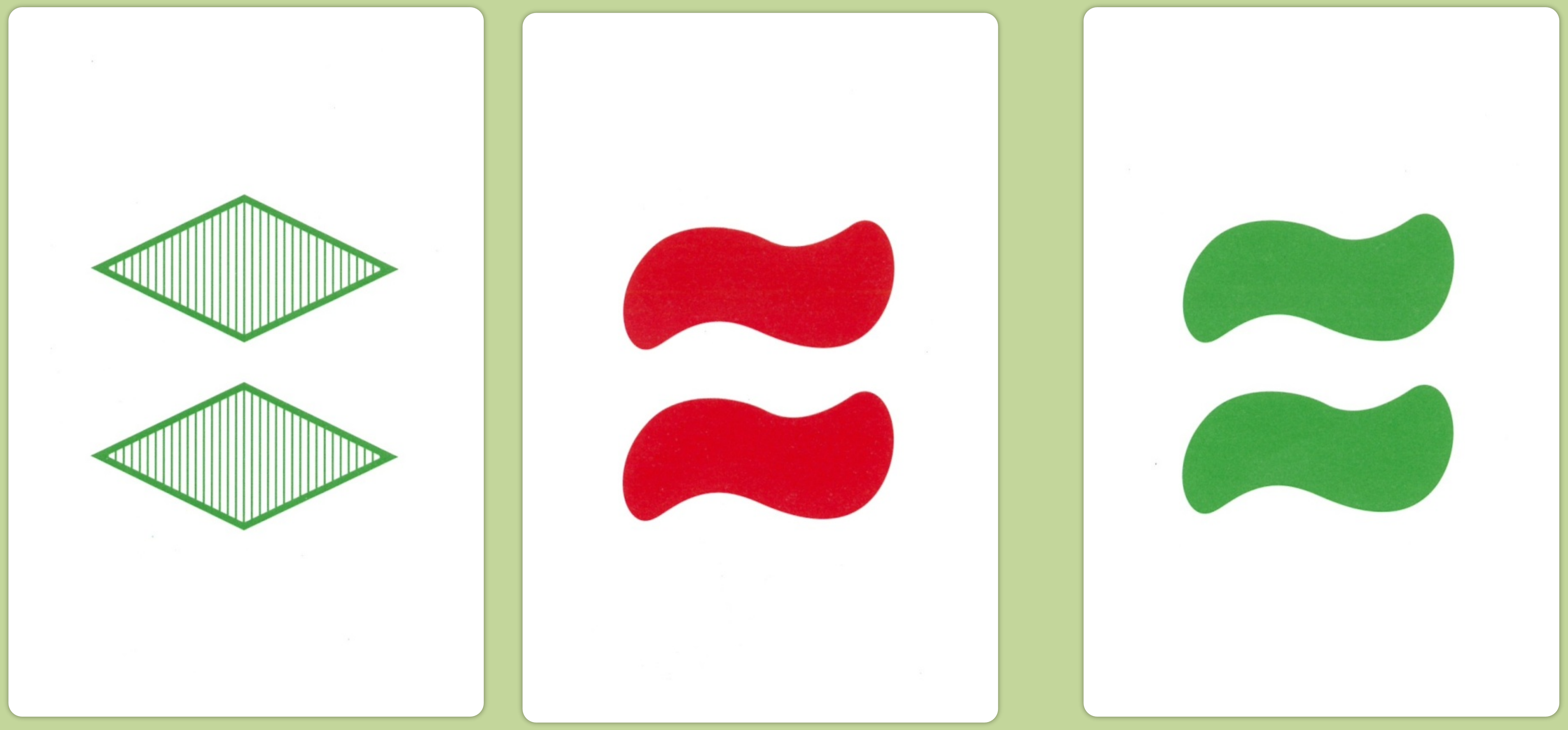
Shading: All the same

Shape: All the same

Yes!

I call this a “two alike Set.”

Is the following a Set?

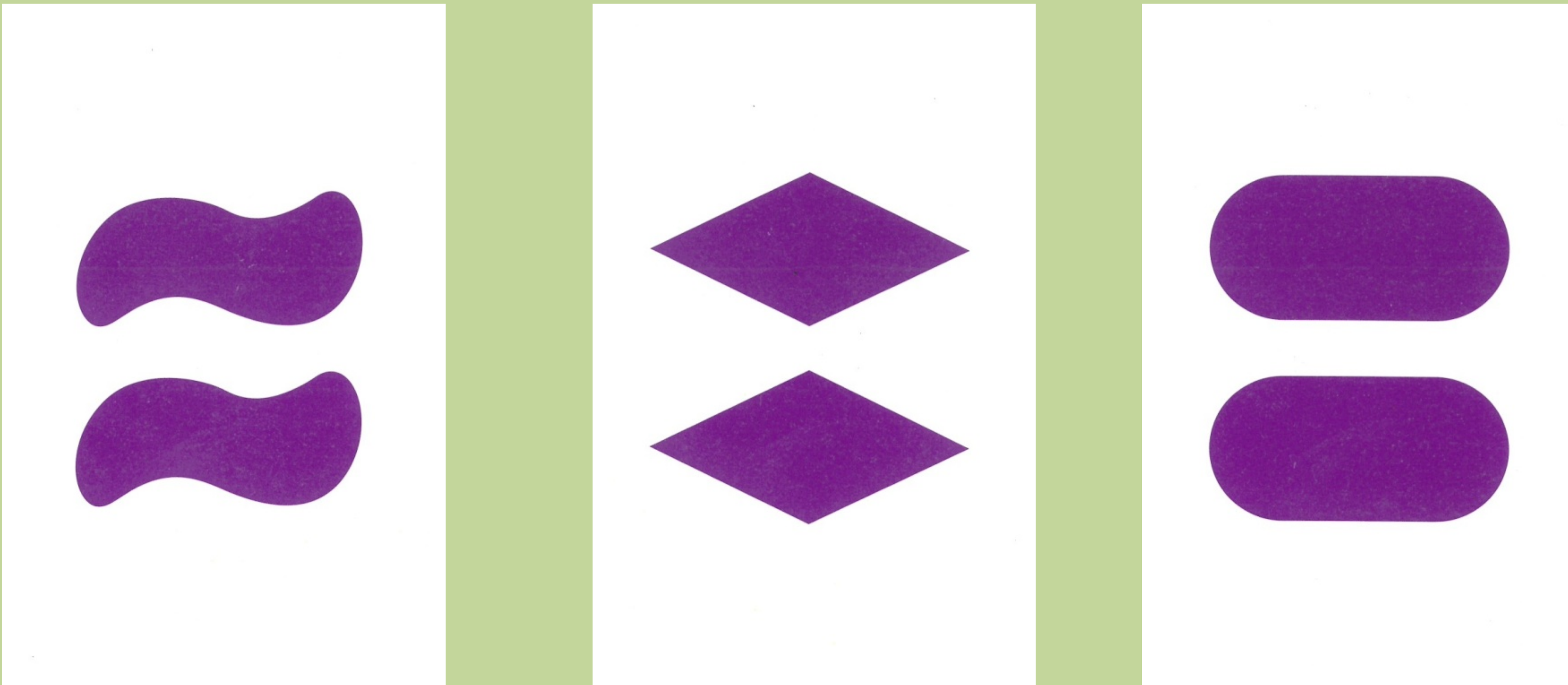


Number: All the same

Shading: Not all the same and not all different

No

Is the following a Set?



Number: All the same

Color: All the same

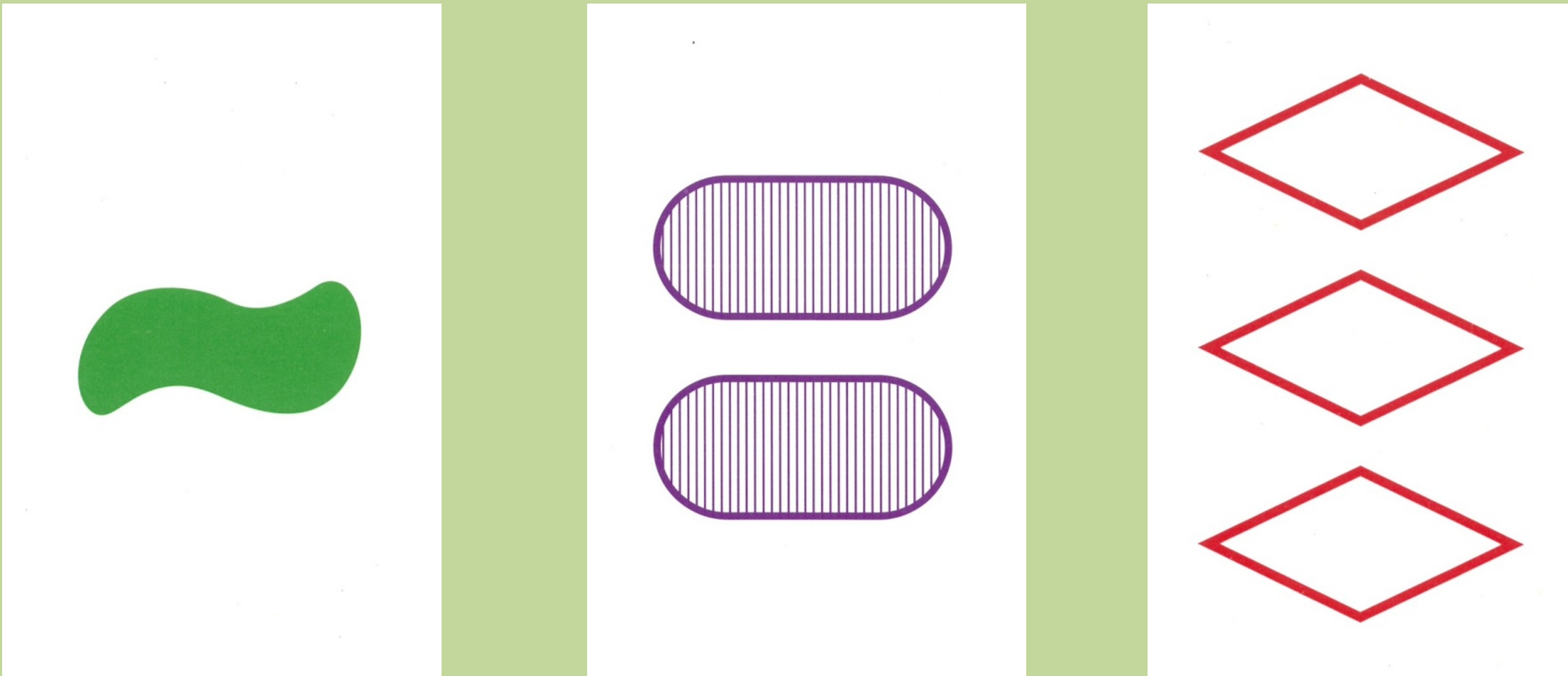
Shading: All the same

Shape: All different

Yes!

I call this a “three alike Set.”

Is the following a Set?



Number: All different

Shading: All different

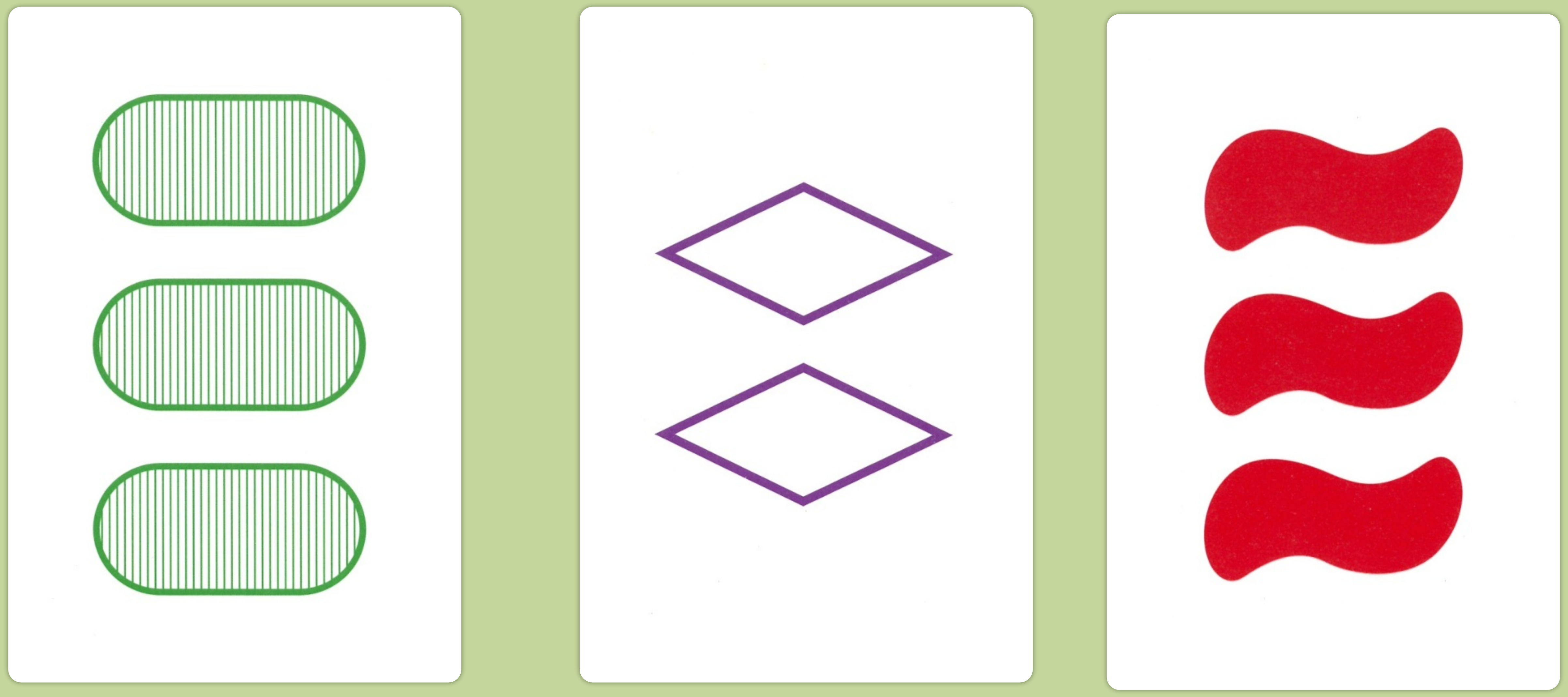
Yes!

Color: All different

Shape: All different

I call this an “all different Set” or a “no alike Set.”

Is the following a set?



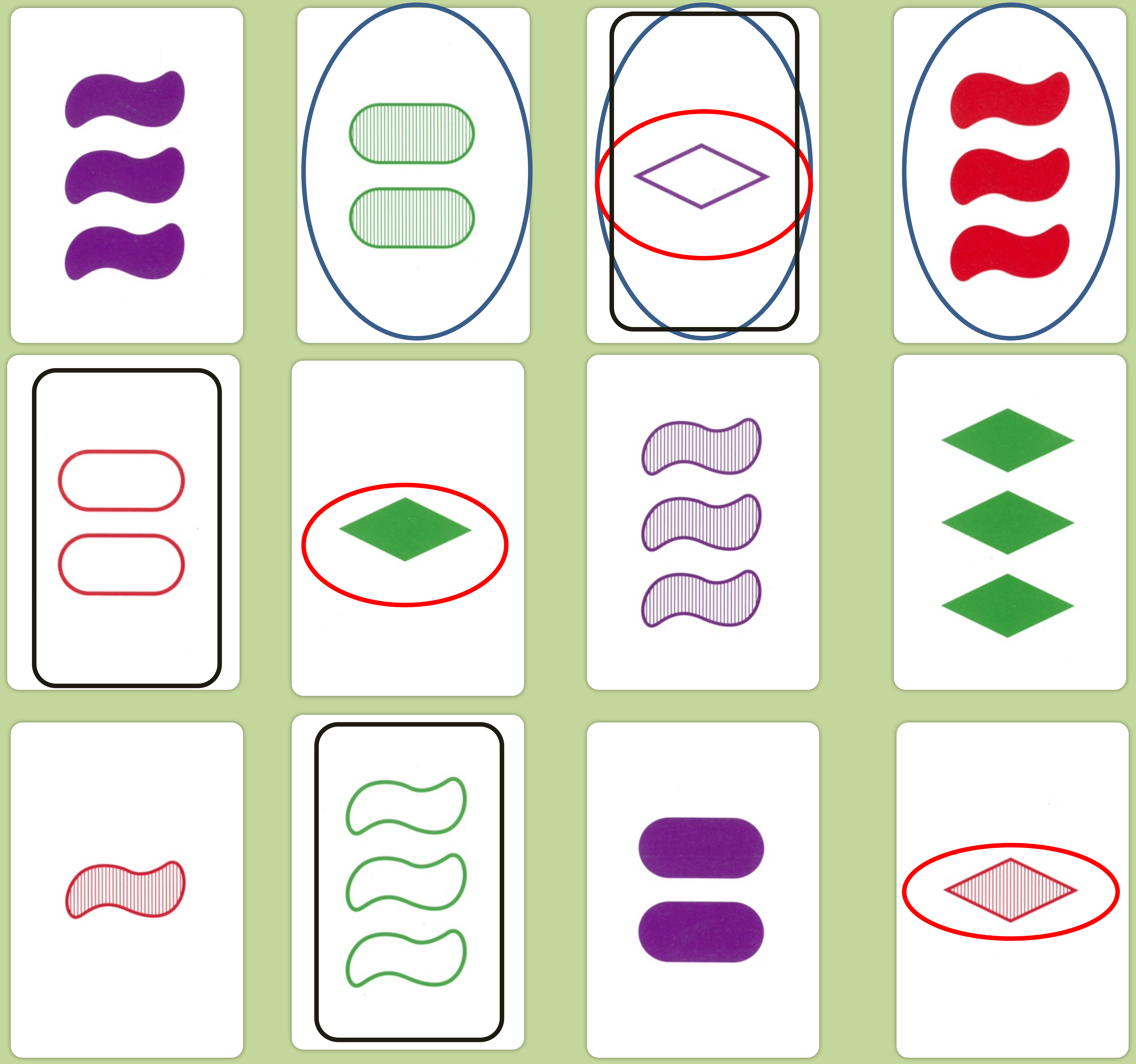
Number: Not the same and
not all different

No!

Playing SET

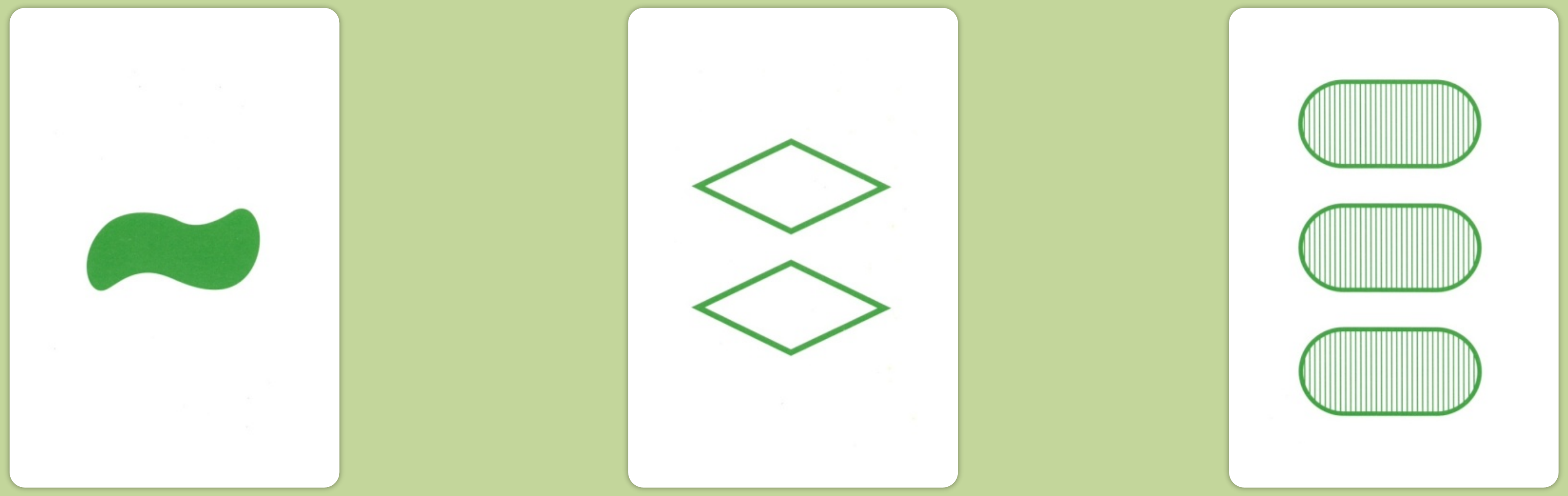
The game is played by dealing an array of 12 cards in the middle of a group of players. The first one to spot a SET in the array shouts "SET" and collects the three cards; then these cards are replaced and play continues.

The person with the most SETs at the end is the winner.

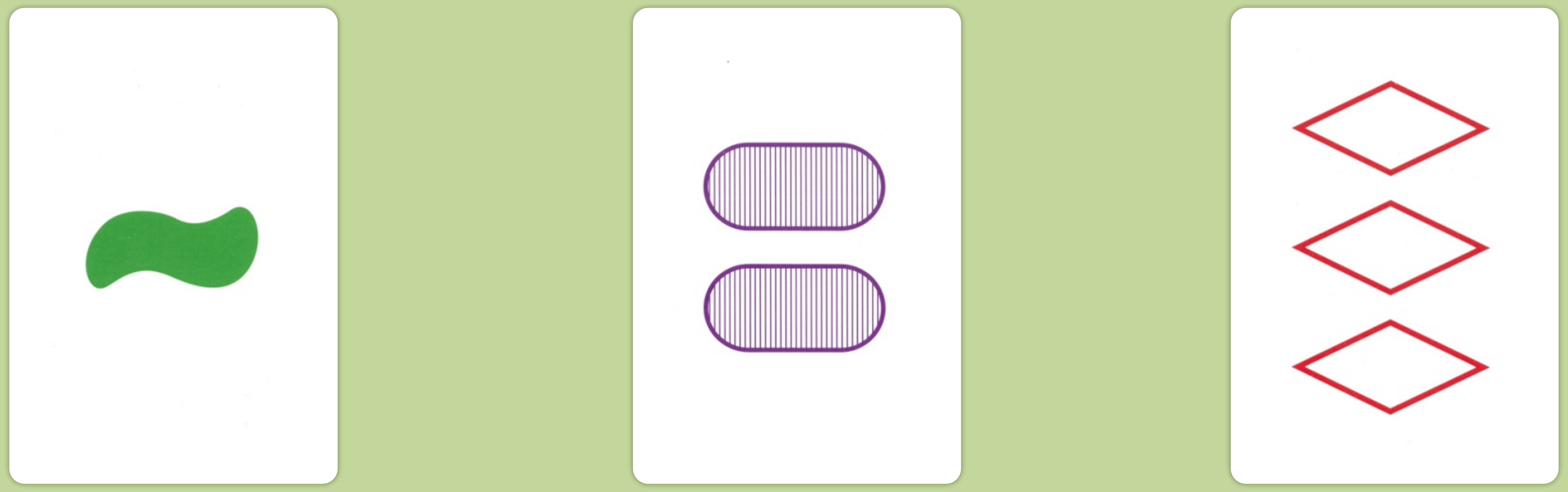


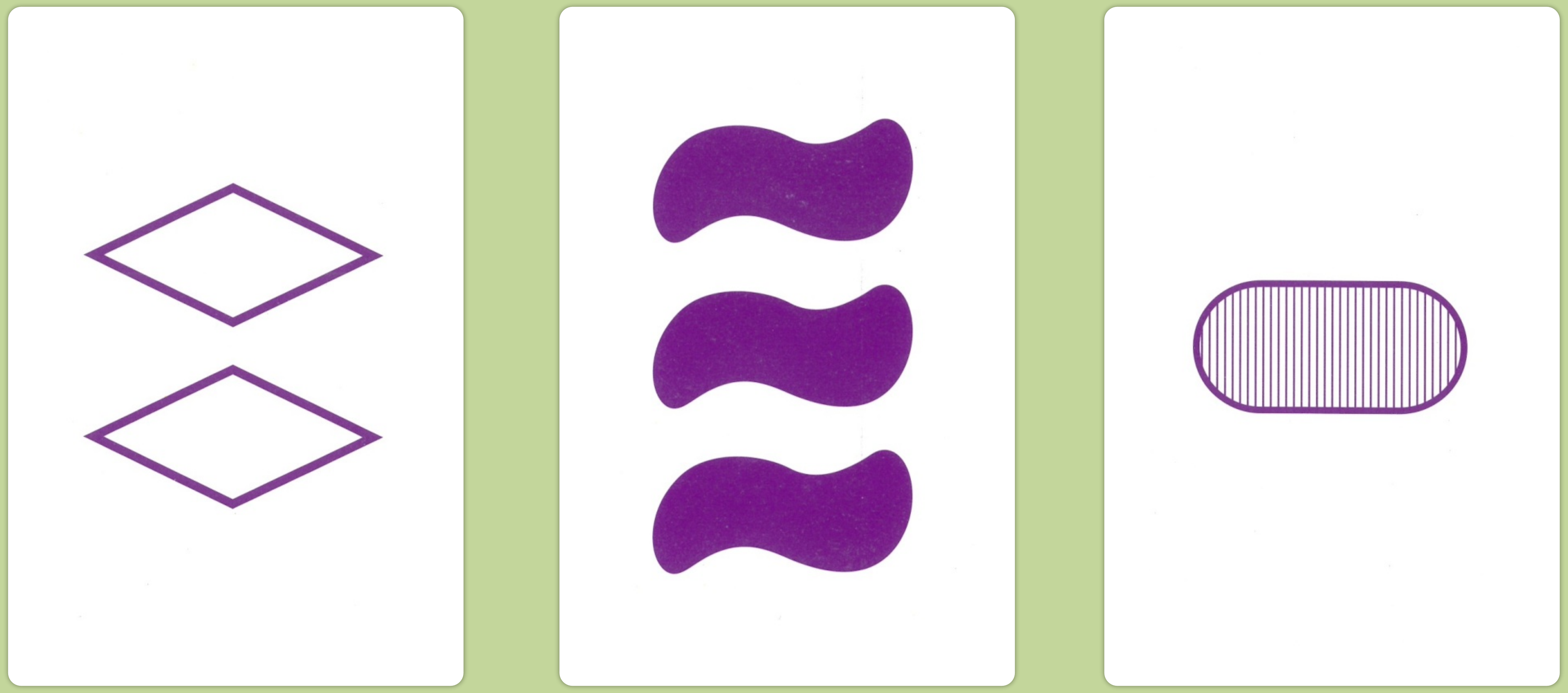
Let’s play Set!!

How many cards does it take to determine a Set?



We can see that one card does not determine a Set. What about two cards?



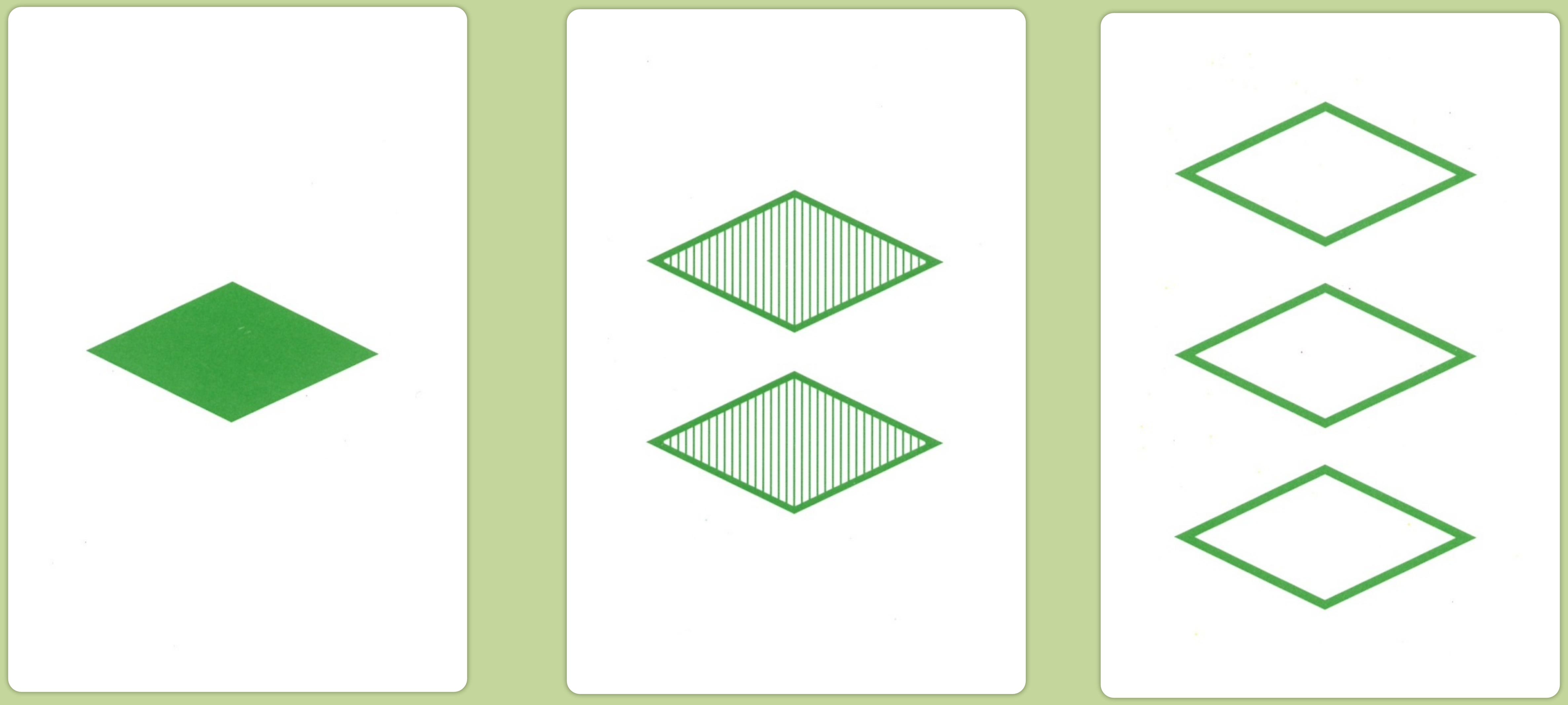


Number: different so third card must be one.

Shading: different so third card must be striped.

Color: all the same so third card must be purple.

Shape: different so third card must be oval.

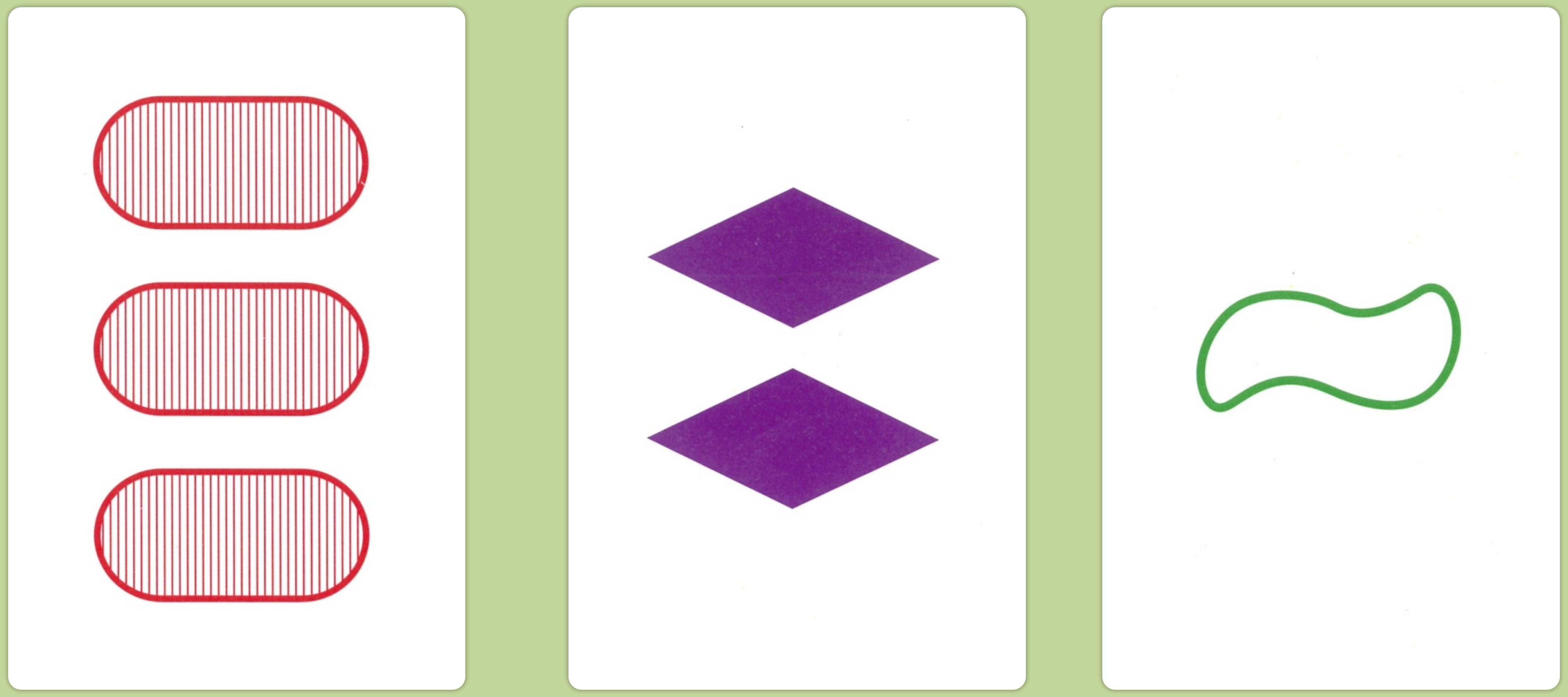


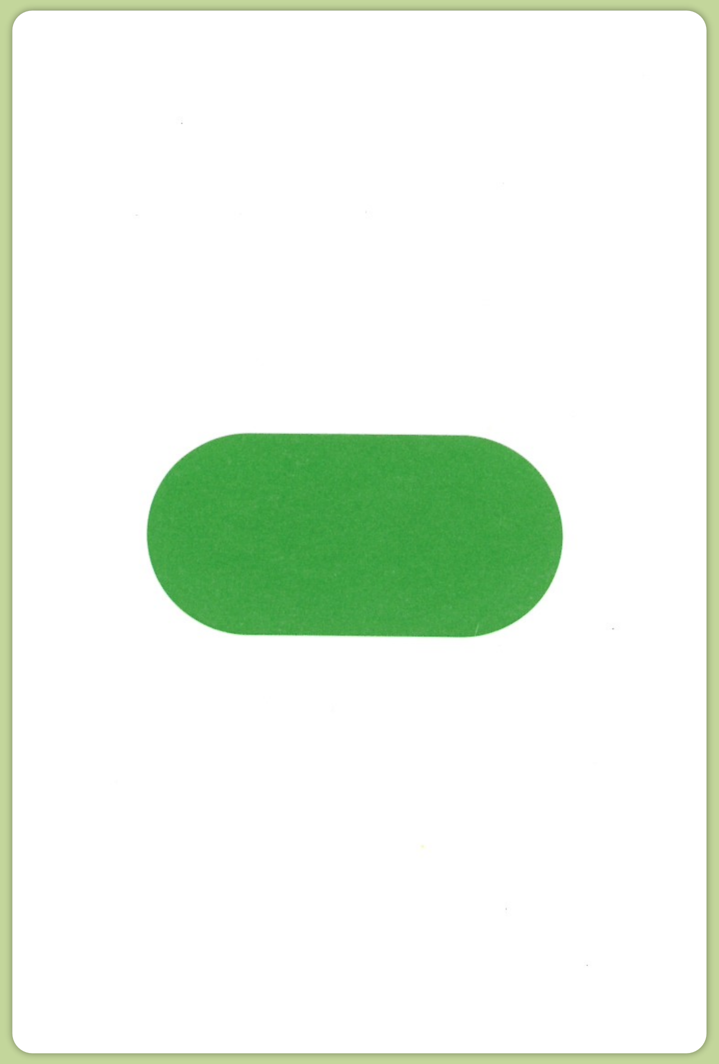
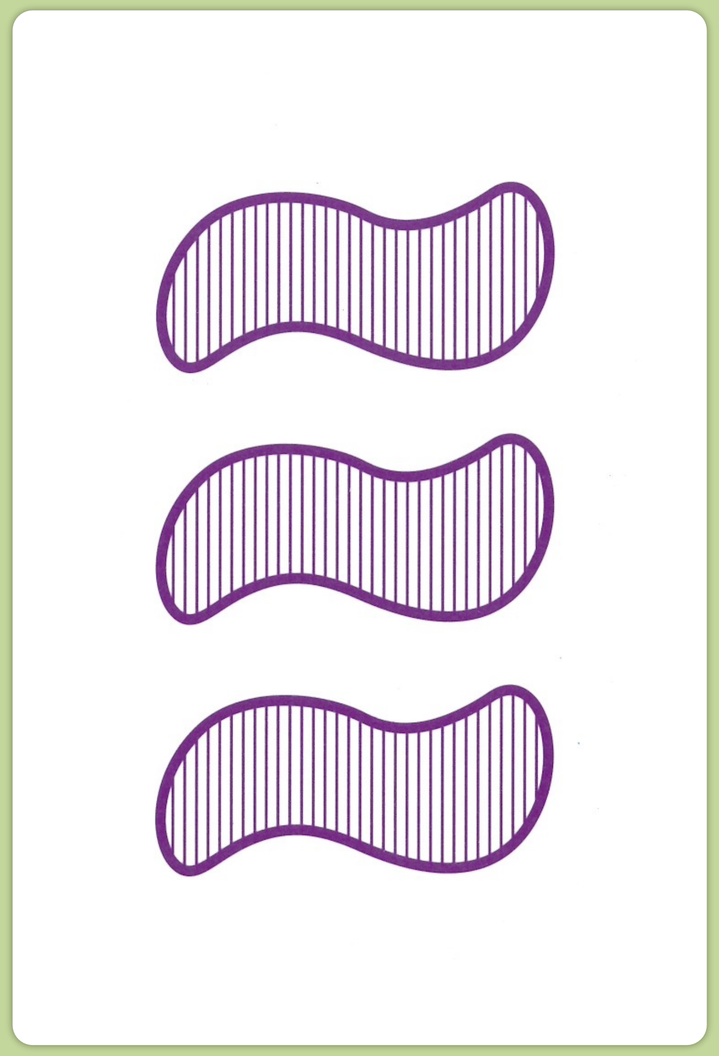
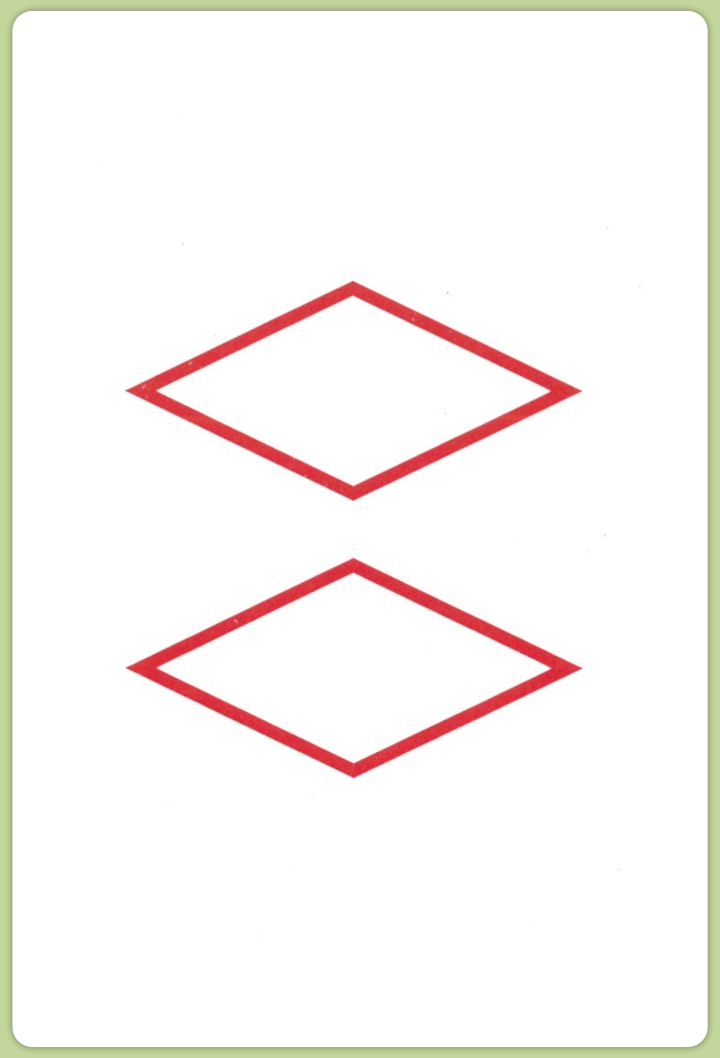
Number: different so third card must be three.

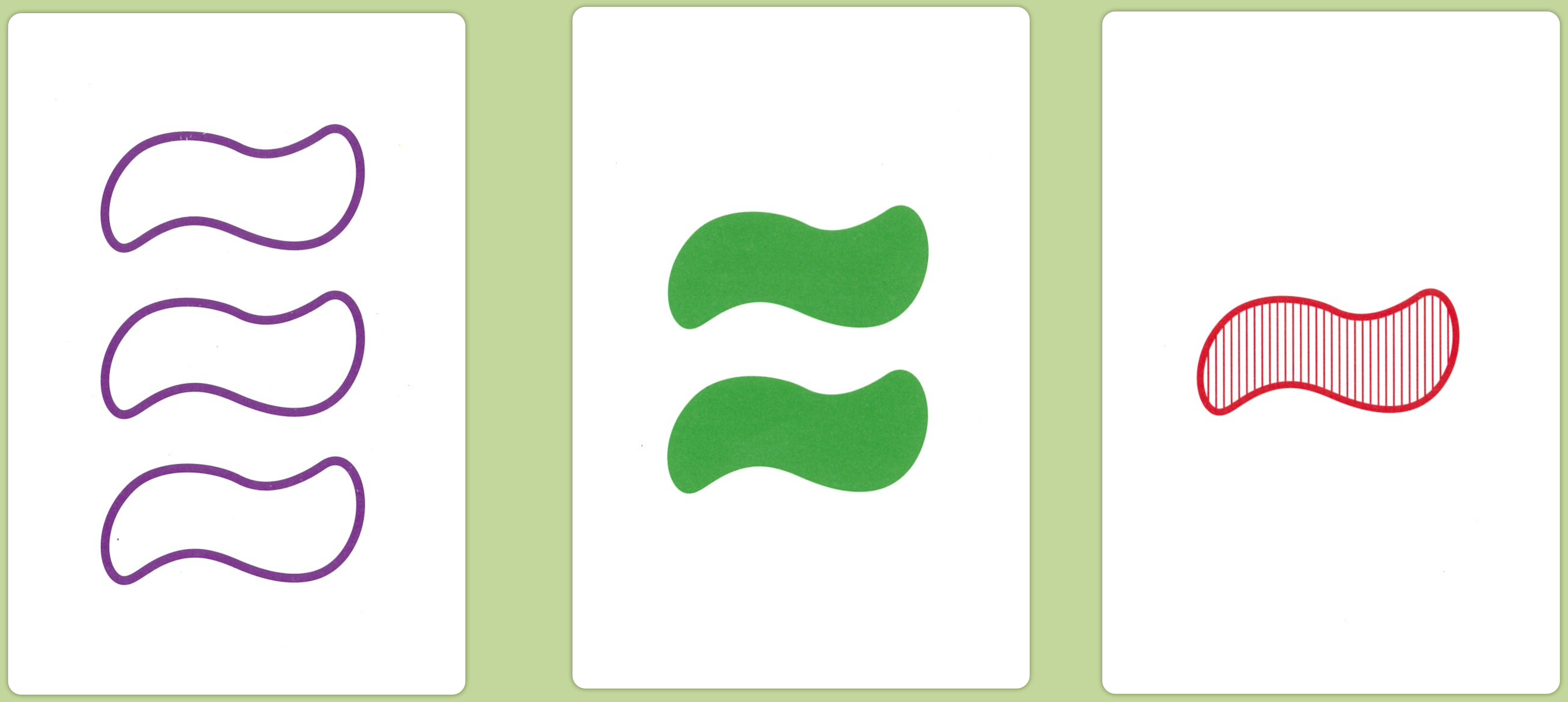
Shading: different so third card must be plain.

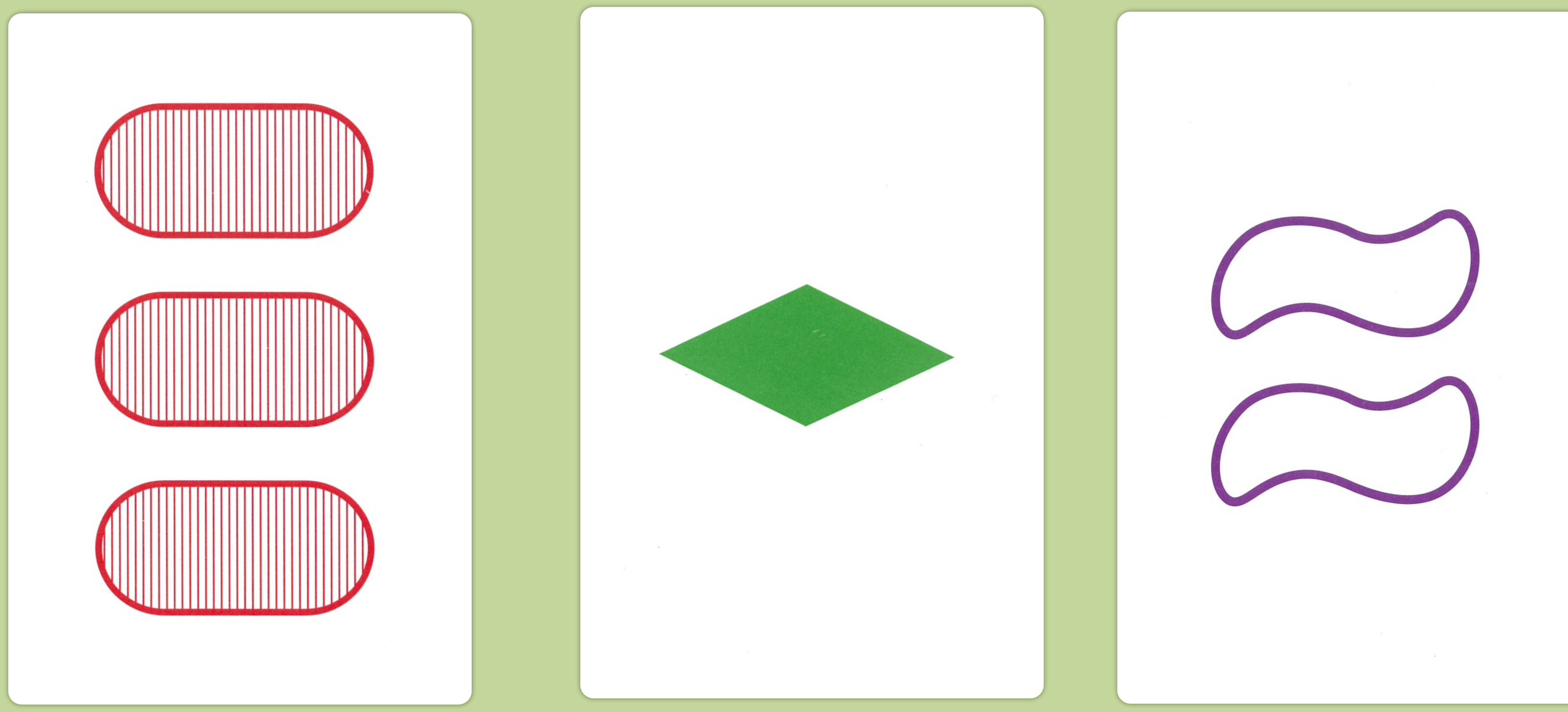
Color: all the same so third card must be green.

Shape: all the same so third card must be diamond.







Will two cards always determine a Set?

Look at each attribute of the first two cards.

If an attribute is the same in both cards, the third card must have this same attribute.

If the first two cards are different in this attribute, then the third card must be different as well.

Therefore, each attribute on the remaining card is uniquely determined.

Let’s play Set!!

What is the probability that three cards drawn at random form a Set?

Draw one card, then draw the second card. How many cards are left? What is the probability that the next card you draw will be the one that makes a Set?

1/79

How many different SETs are there?

How many different SETs are there?

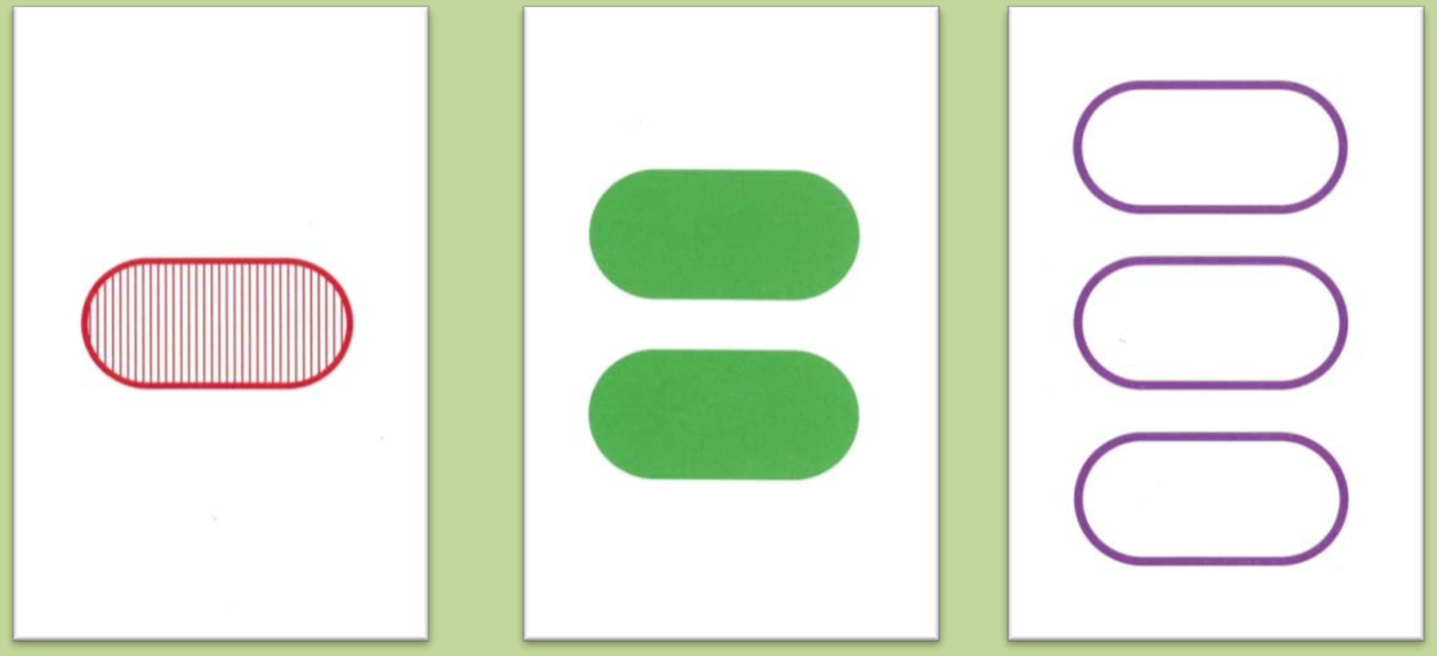
81 x 80 x 1

There are 81 possibilities for the first card in the SET.

There are 80 possibilities for the second card in the SET.

There is then one unique card for the third card to make a SET.

For example, suppose we take One striped red oval for the first card; then Two solid green ovals for the second card; then …



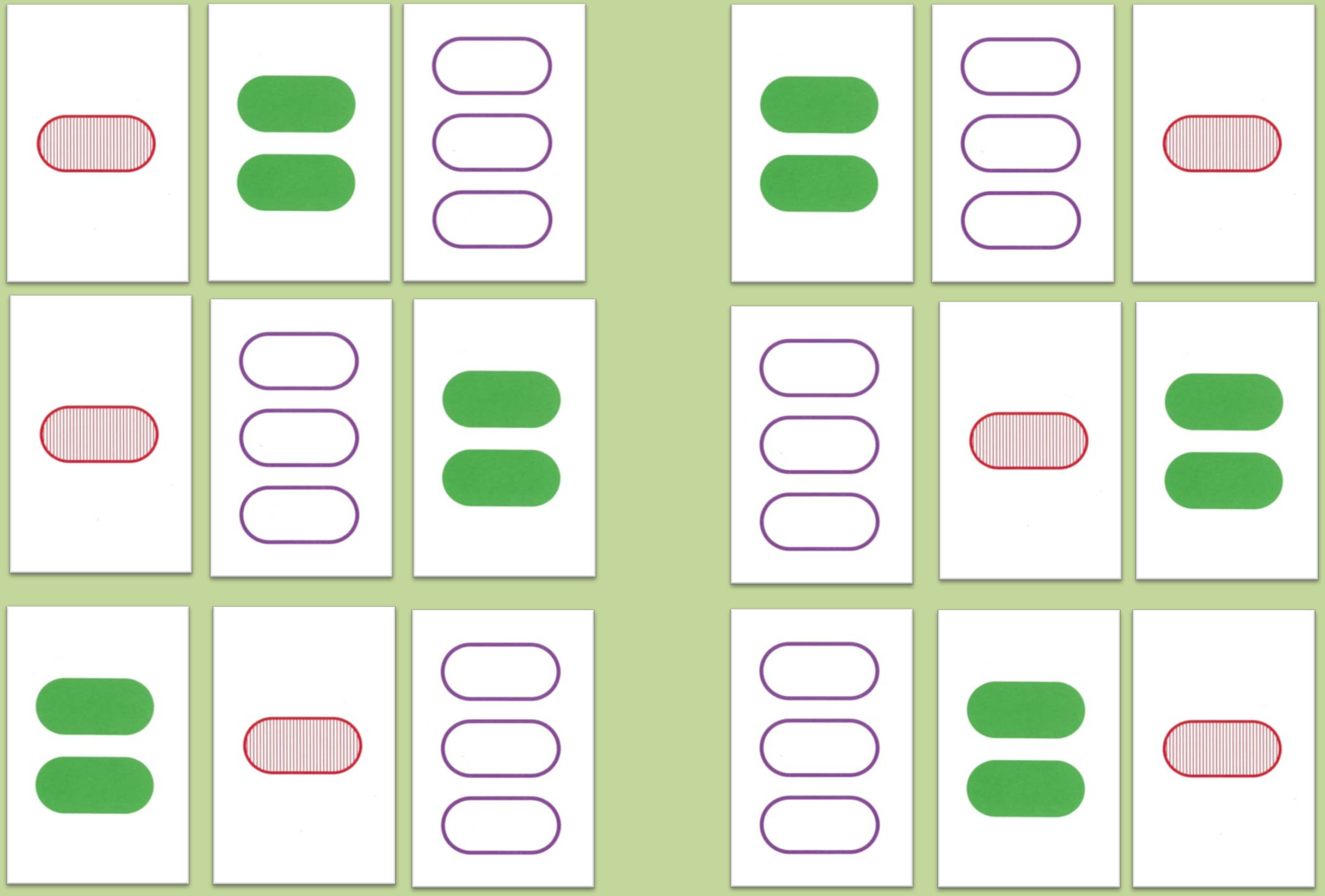
What if, instead, we took Two solid green ovals for the first card; and then Three plain purple ovals for the second card; then …

Clearly this is the same SET as before;
it has been (at least) double counted.

How should we correct this formula?

$$\frac{81}{\quad} \times \frac{80}{\quad} \times \frac{1}{\quad}$$

It has actually been counted 6 times



How many different SETs are there?

We need to divide by 3! = 3 x 2 x 1

$$\frac{81 \times 80 \times 1}{3 \times 2 \times 1}$$

How many different SETs are there?

$$\frac{81 \times 80 \times 1}{3 \times 2 \times 1} = 27 \times 40 = 1080$$

There are 1080 different SETs.

Questions to Ponder

- What types of sets are there?
- Are some types of sets easier to identify than others?
- Are there more of some types of sets than others?
- Of the types of sets, which type occurs most often?
- Do the harder to identify sets occur less often than the other types of sets?

Types of Sets

There are actually four different kinds of SETs. They can be classified according to how many characteristics the cards share: zero, one, two, or three.

The percentage of each type of set can be easily calculated.

In geometry, there are three undefined terms. These terms are **point, line and plane**.

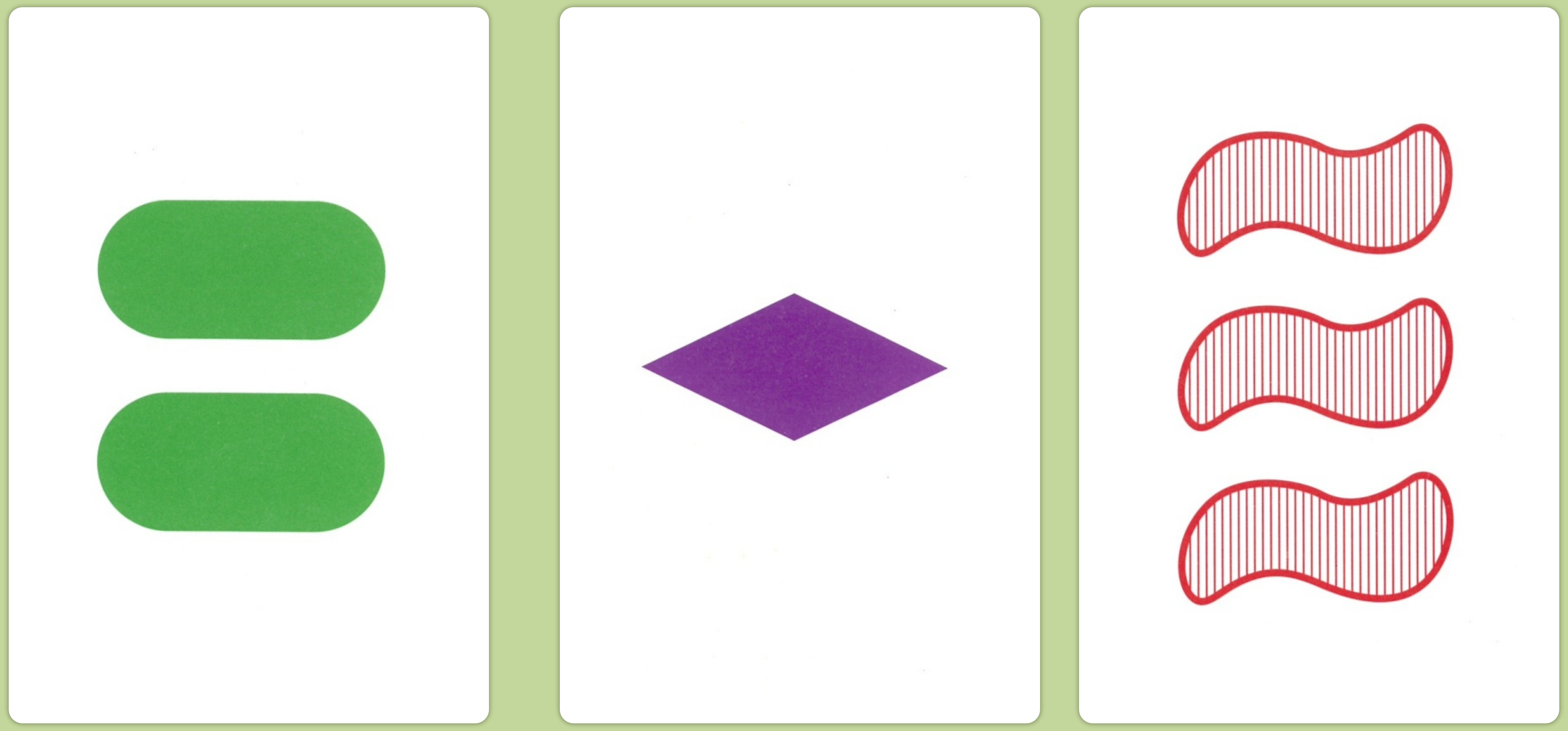
Different models for geometry can be created by defining the above terms. Some of these models are finite and some are infinite.

Define point to be an individual Set card.

Define line to be a collection of three points that form a Set.

How should we define Plane?

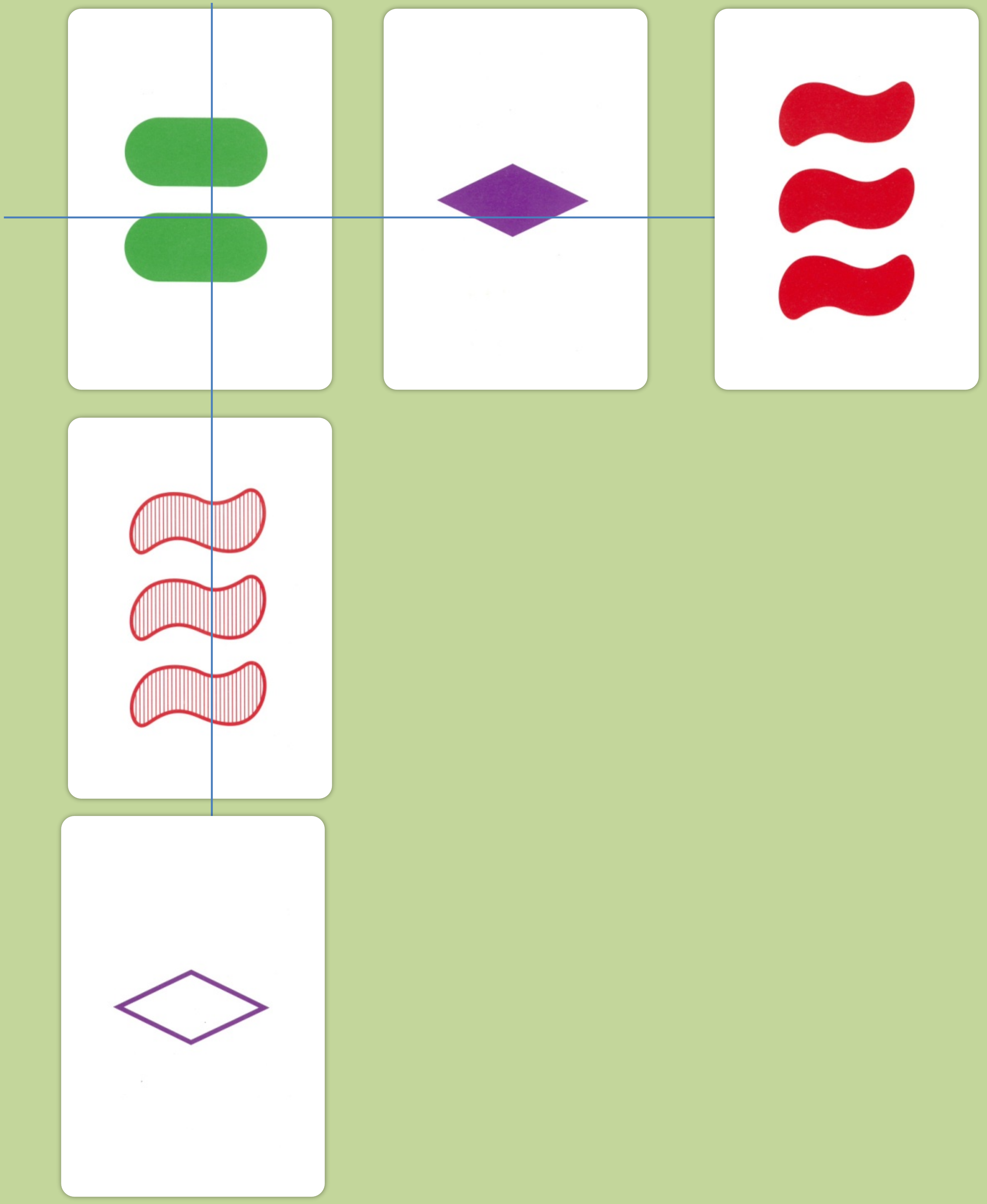
Three non-collinear points determine a plane, so lets start with three non-collinear points.

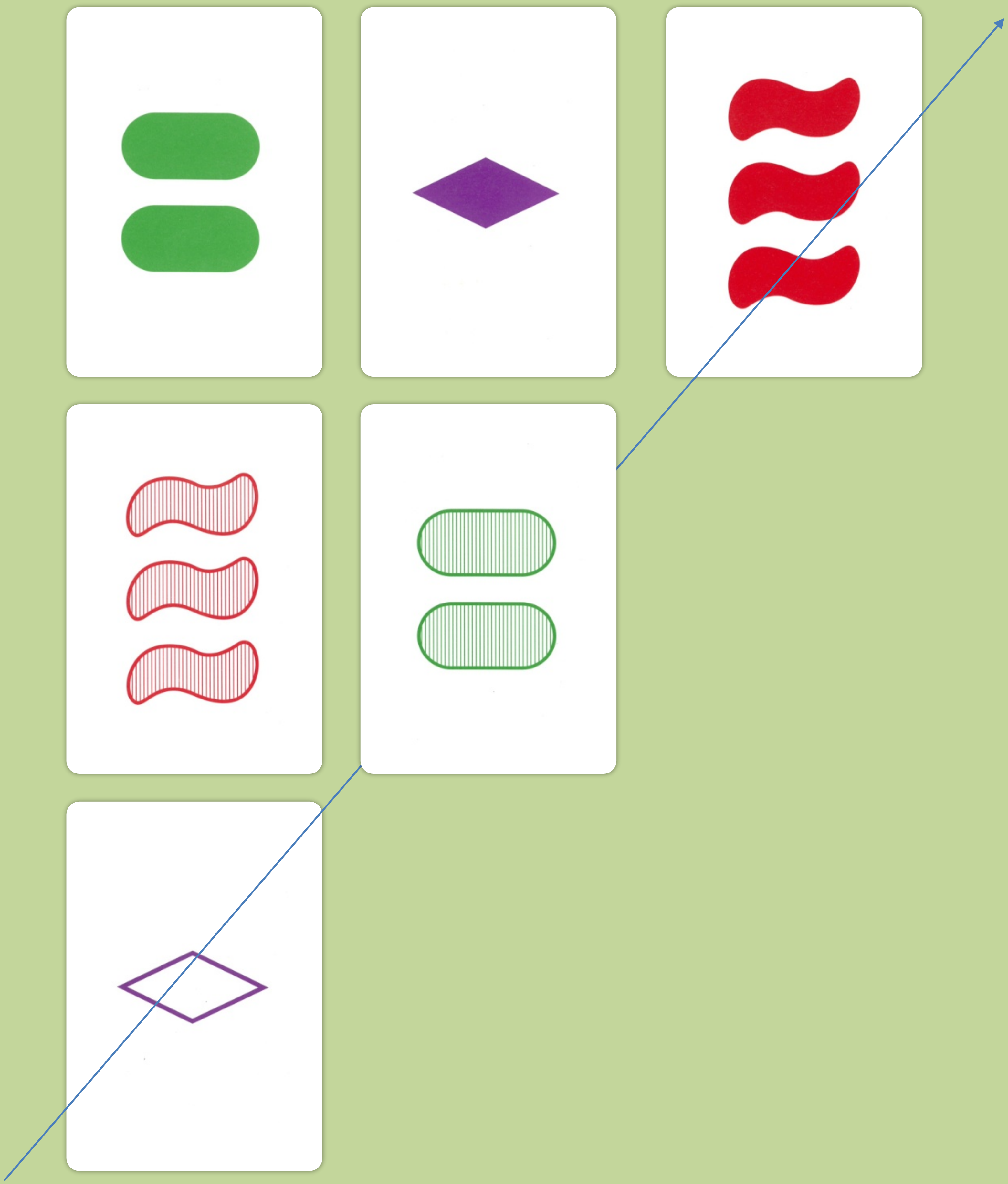


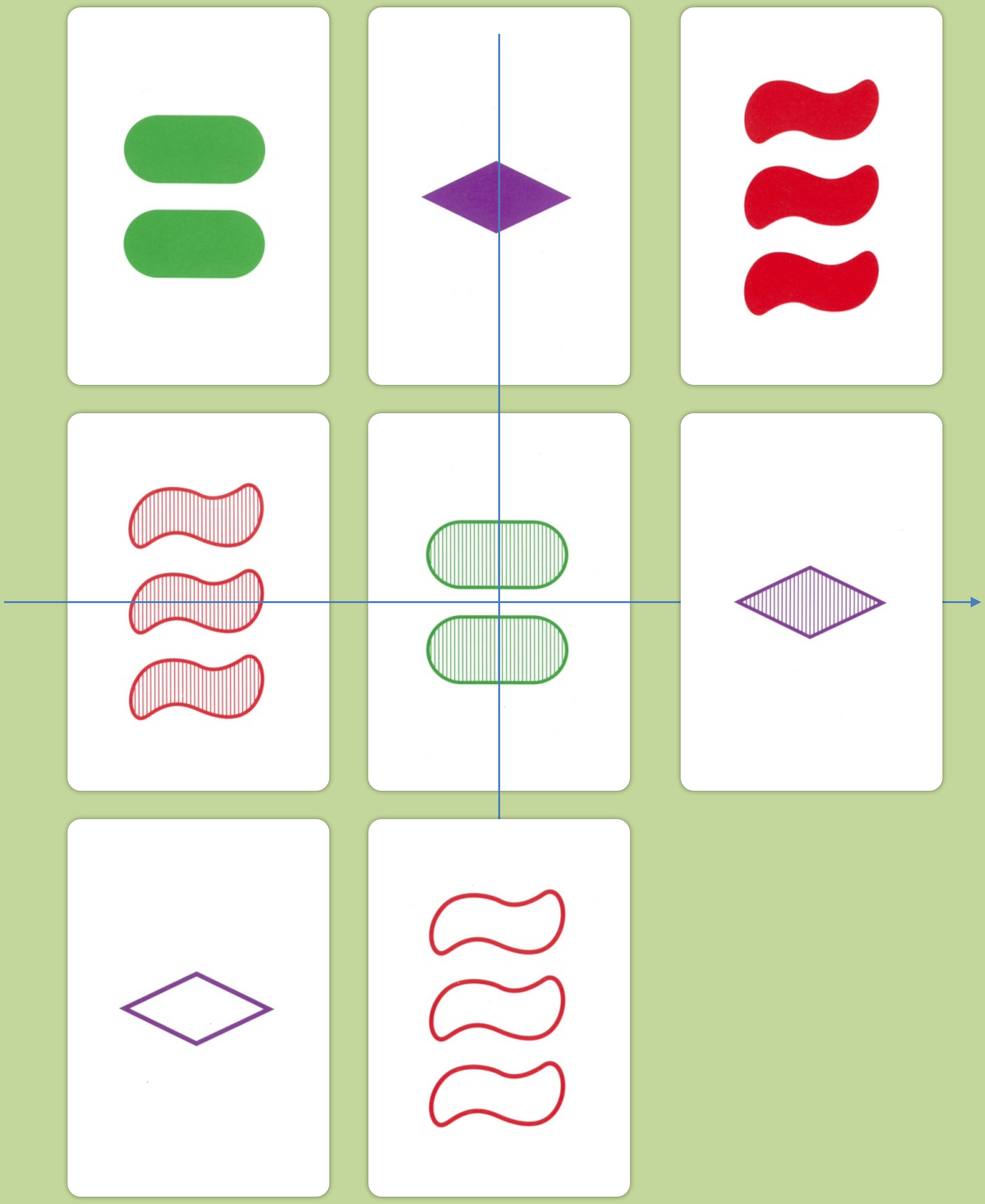
“If two points lie in a plane, then any line containing those two points also lies in that plane.”

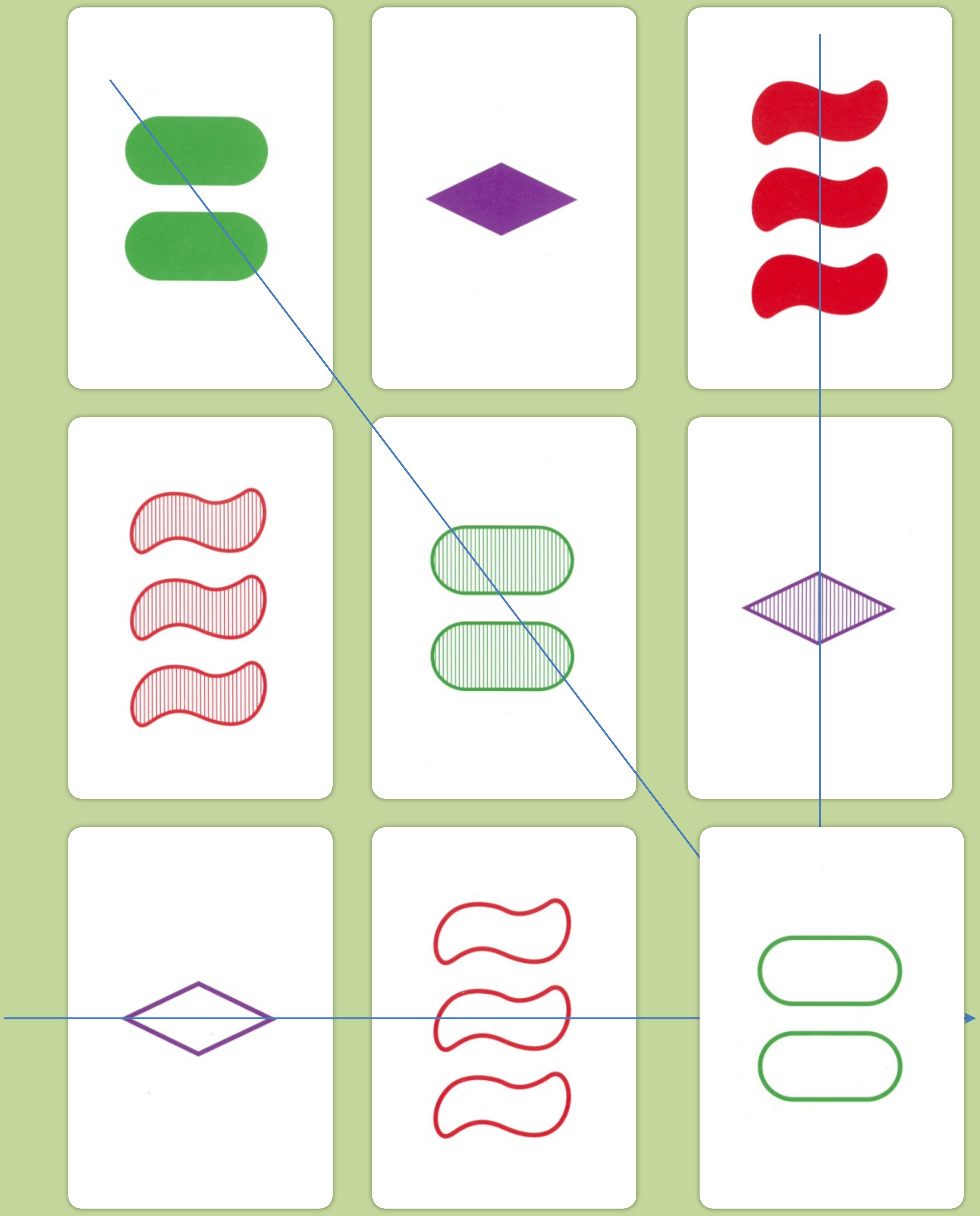
This means that for any two points in our plane, the line (Set) determined by those points should lie in the plane. This means that the third card required to complete the Set should also be a part of the plane.

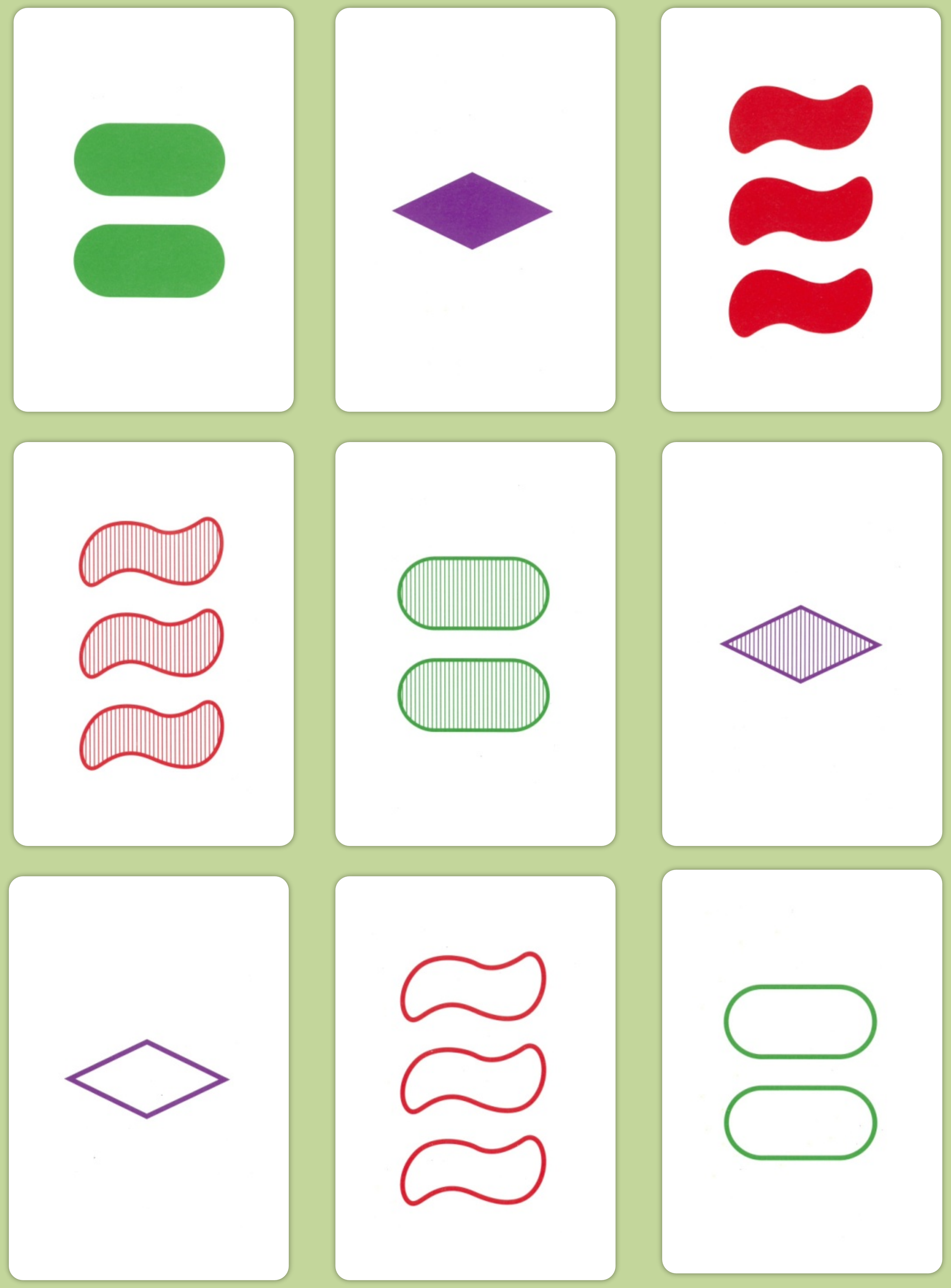
Arrange the cards in an upside down L and start completing lines (Sets).





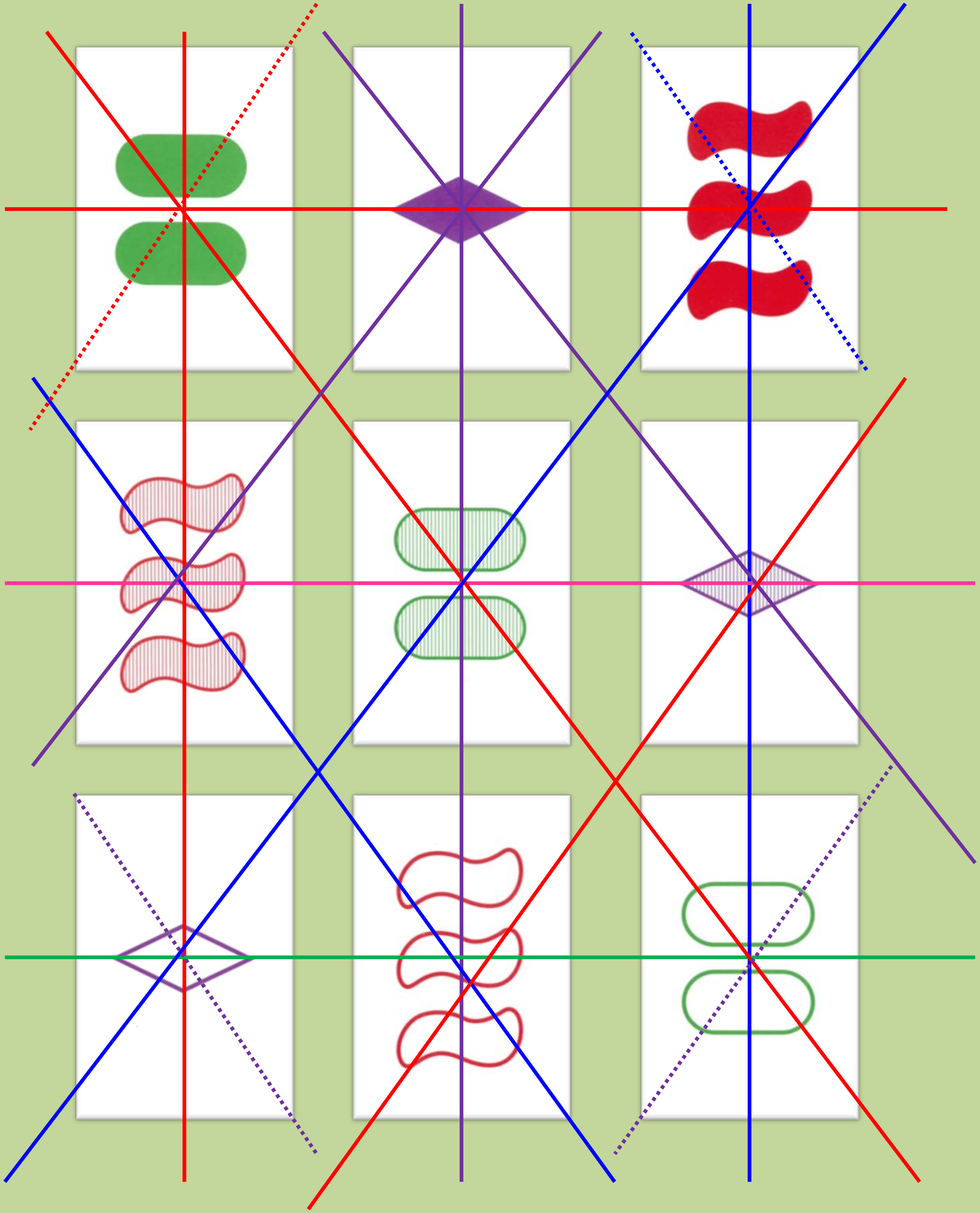




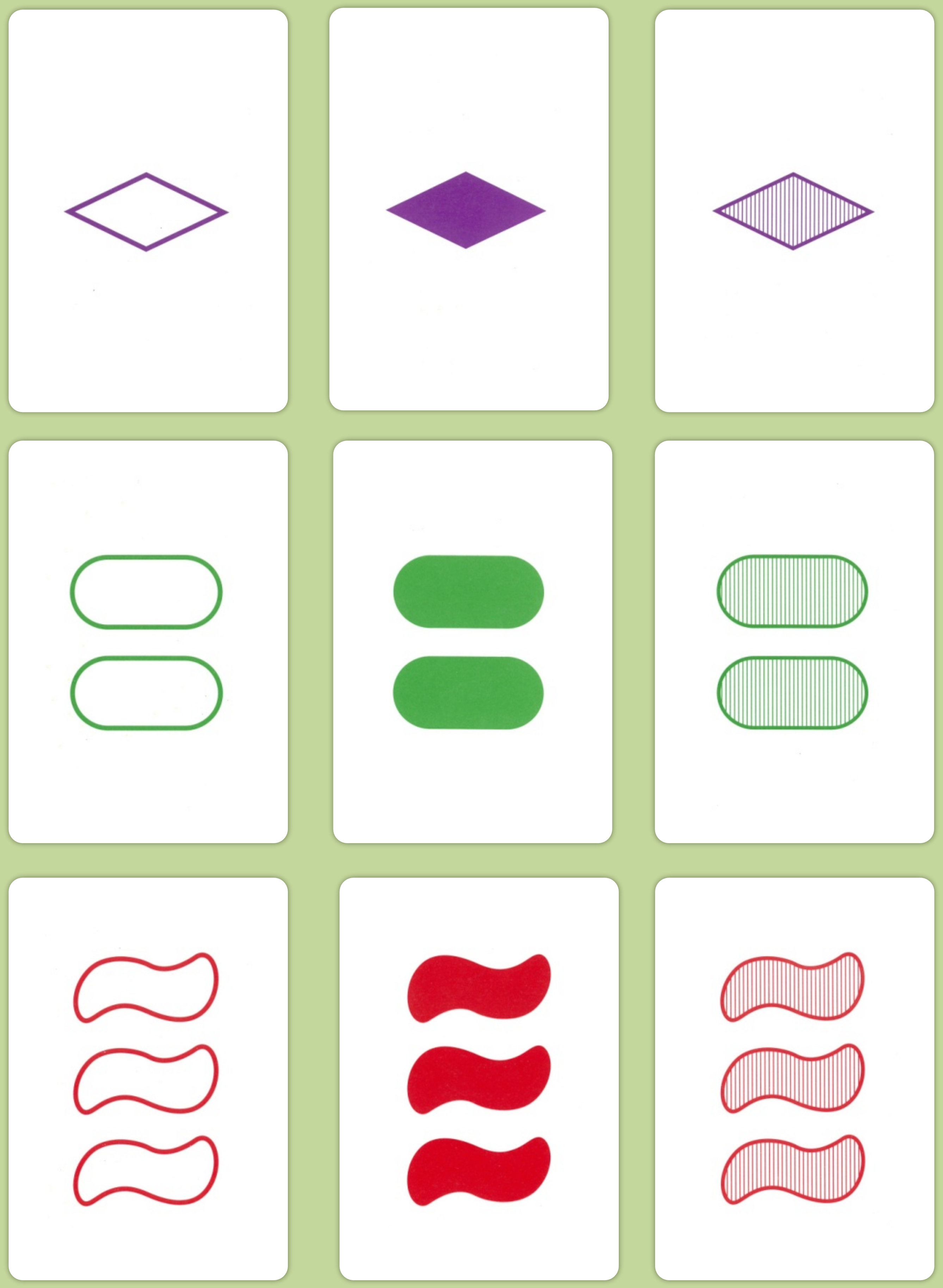


Are there any other cards that should be included?

How many total Sets are in this plane?



In fact, in every case we will end up with a plane containing nine points (cards) and 12 lines (Sets).



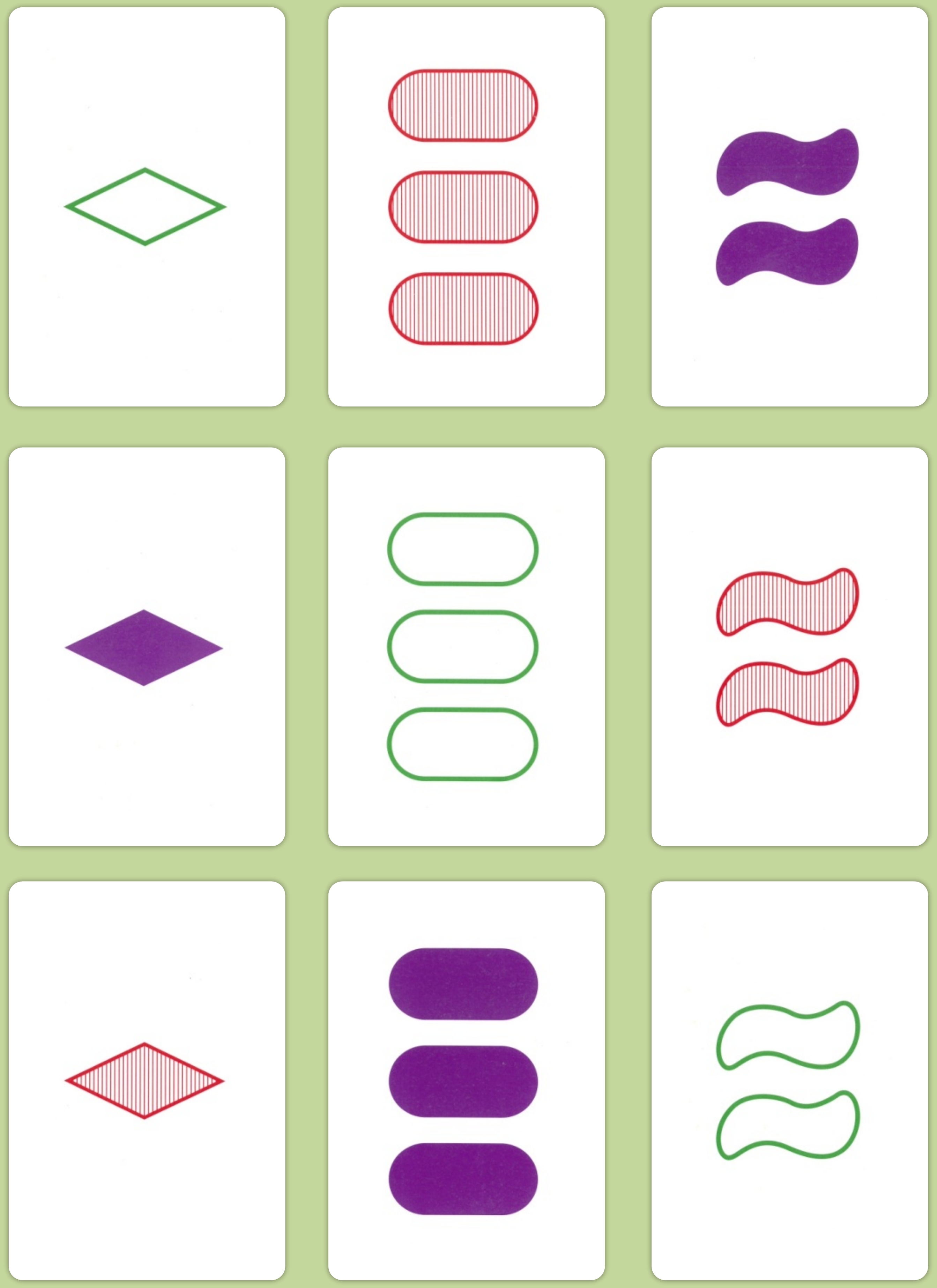
Describe this plane:

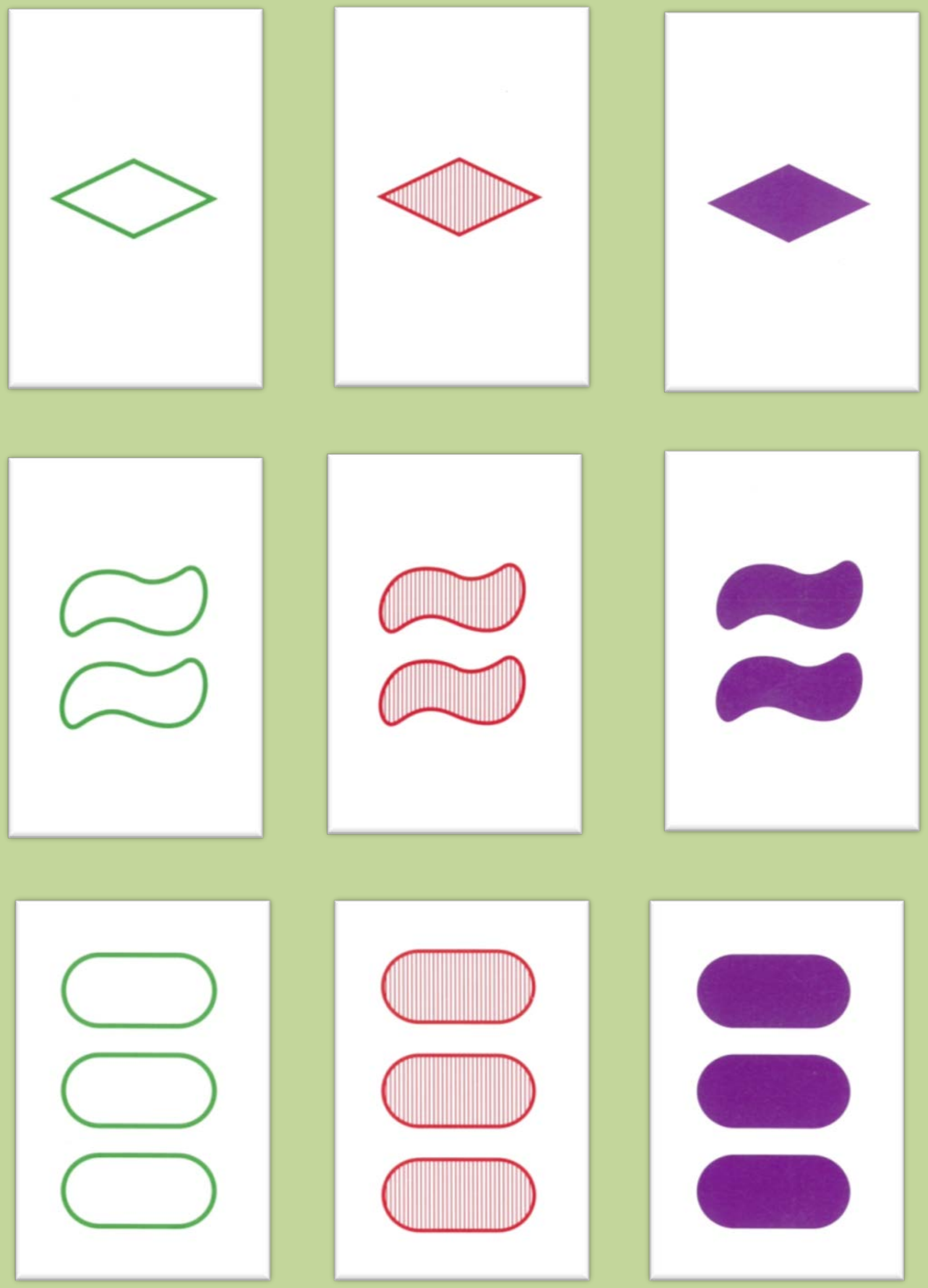
All of the one purple diamonds

All of the two green ovals

All of the three red squiggles

LETS MAKE ANOTHER PLANE!!

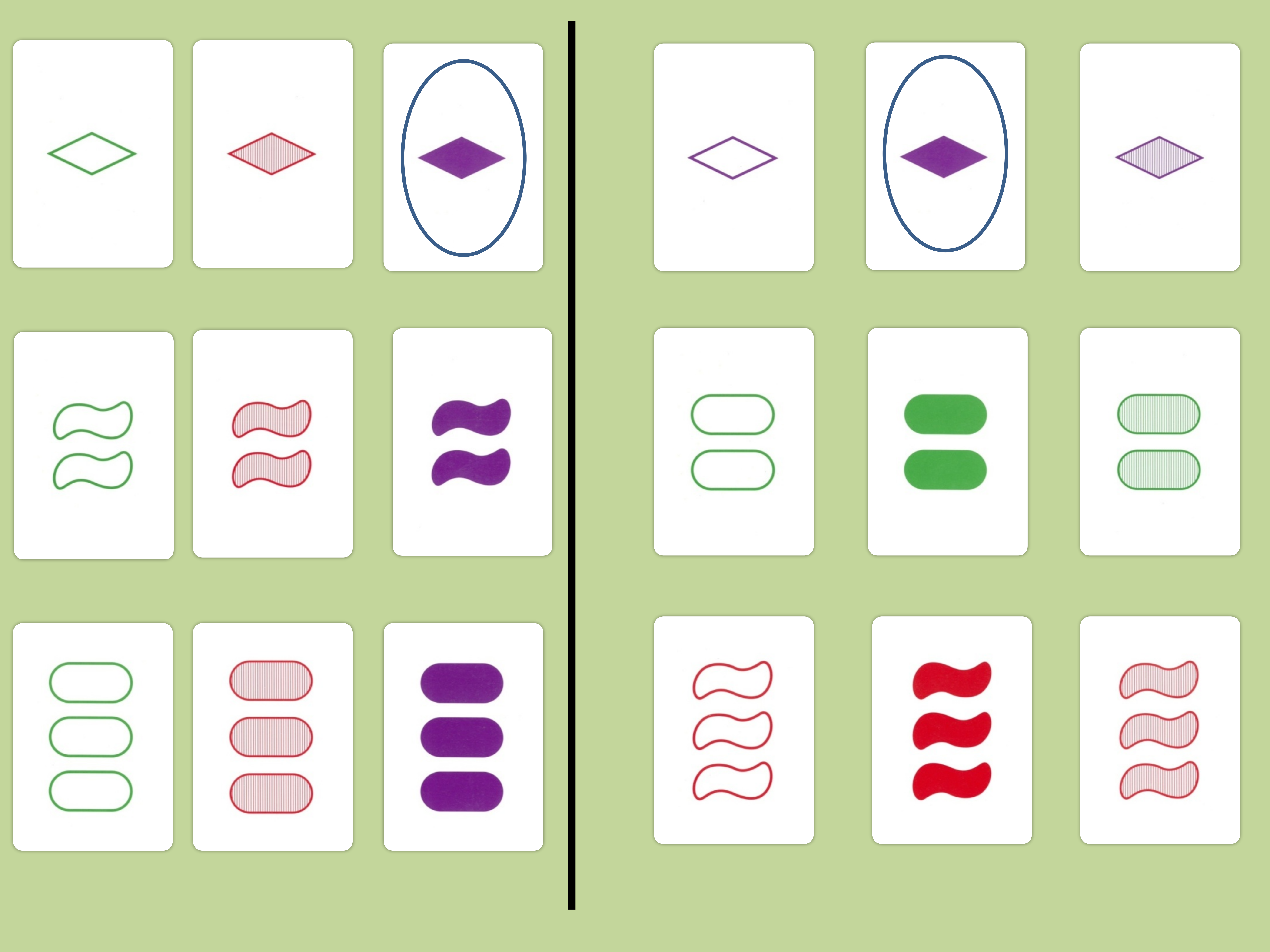




How many distinct planes exist?

Can you classify the different types of planes as we did earlier with the Sets?

What is the intersection of our two planes?



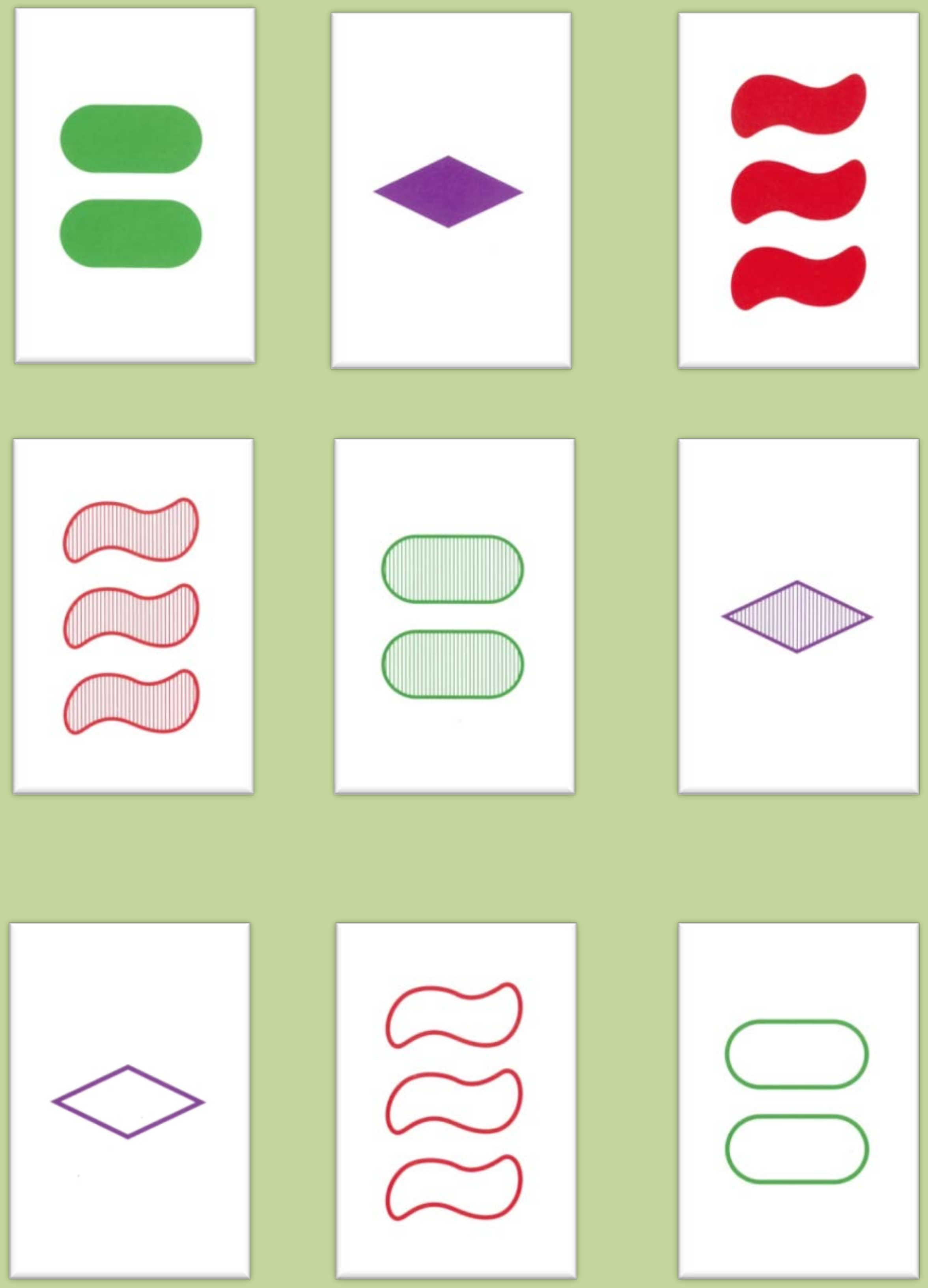
To get a plane, we started with three non-collinear points (cards). We then included additional points to make sure that if two points(cards) were in the plane then the entire line(Set) determined by those two points(cards) was in the plane.

We can extend this idea in our Set geometry model to create hyperplanes. We will start with four non-coplanar points and add points(cards) to make sure we have all lines (Sets).

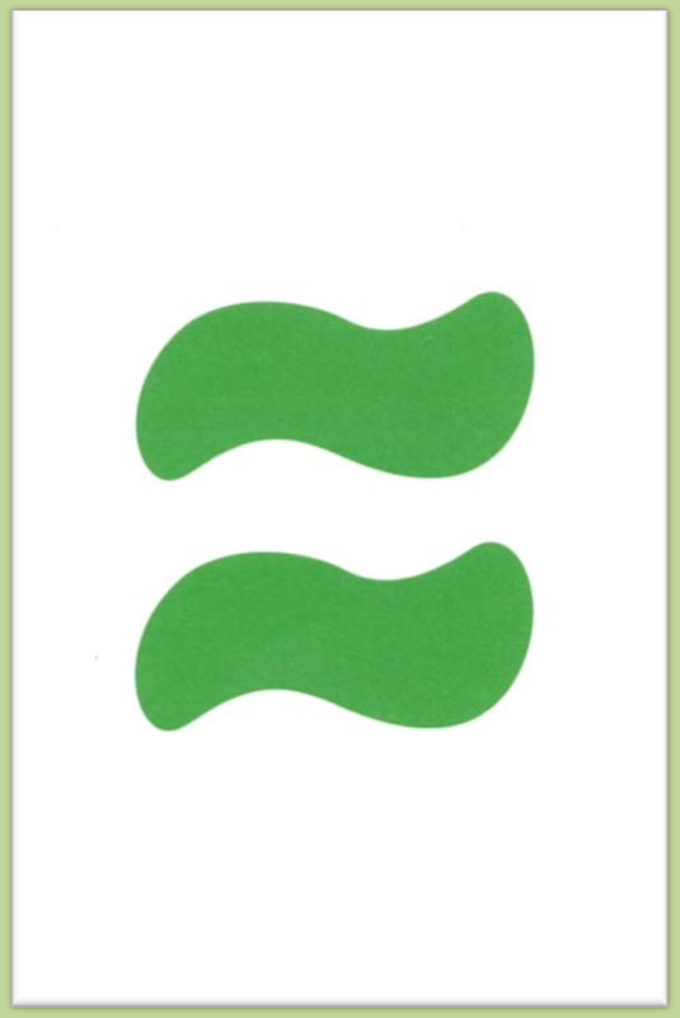
How do we determine if four points are non-coplanar?

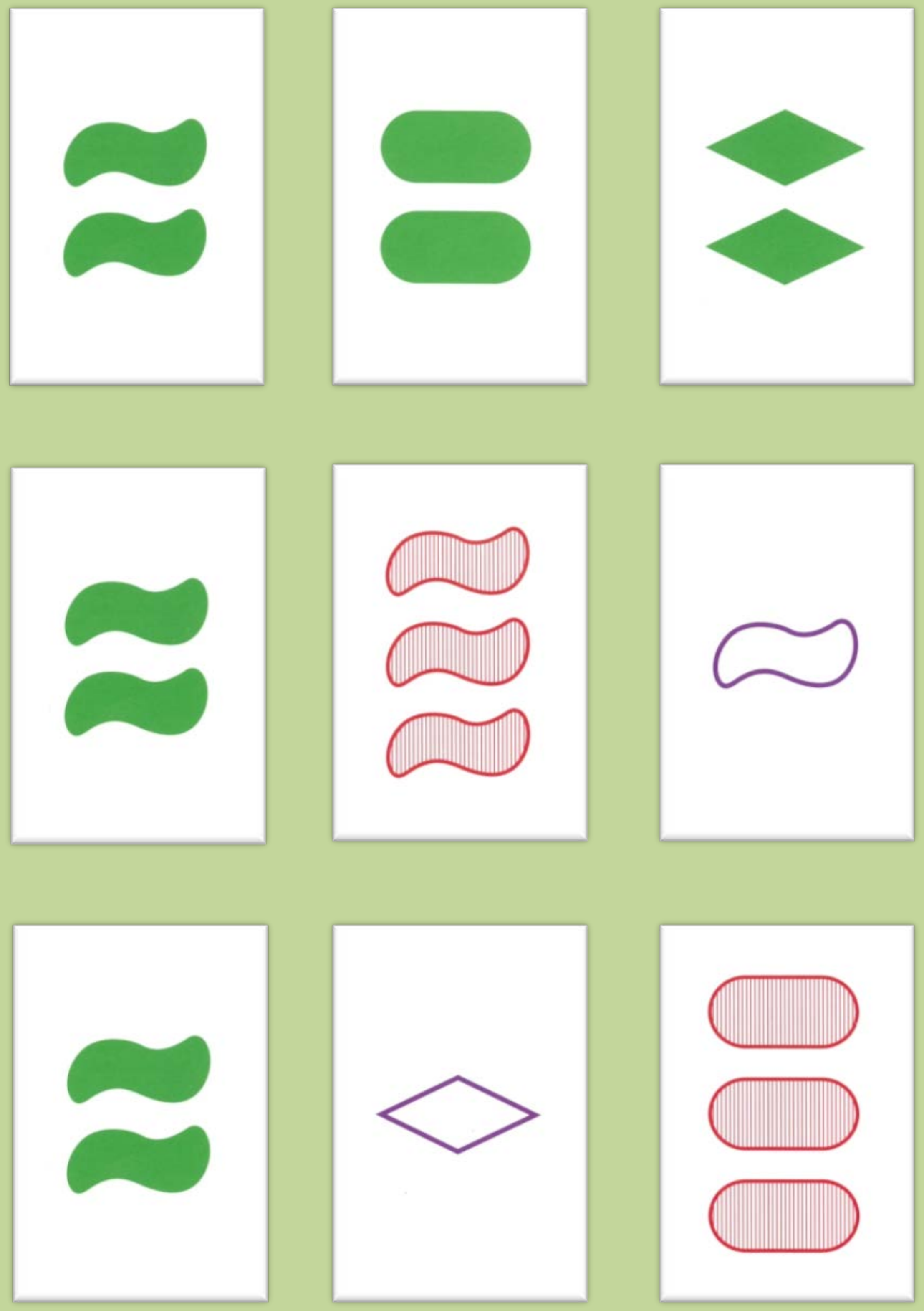
Once we have 3 non-collinear points we see that the remaining points in the plane have been determined, so we need a point that will not be in the plane.

Start with an entire plane and then add one additional point(card).

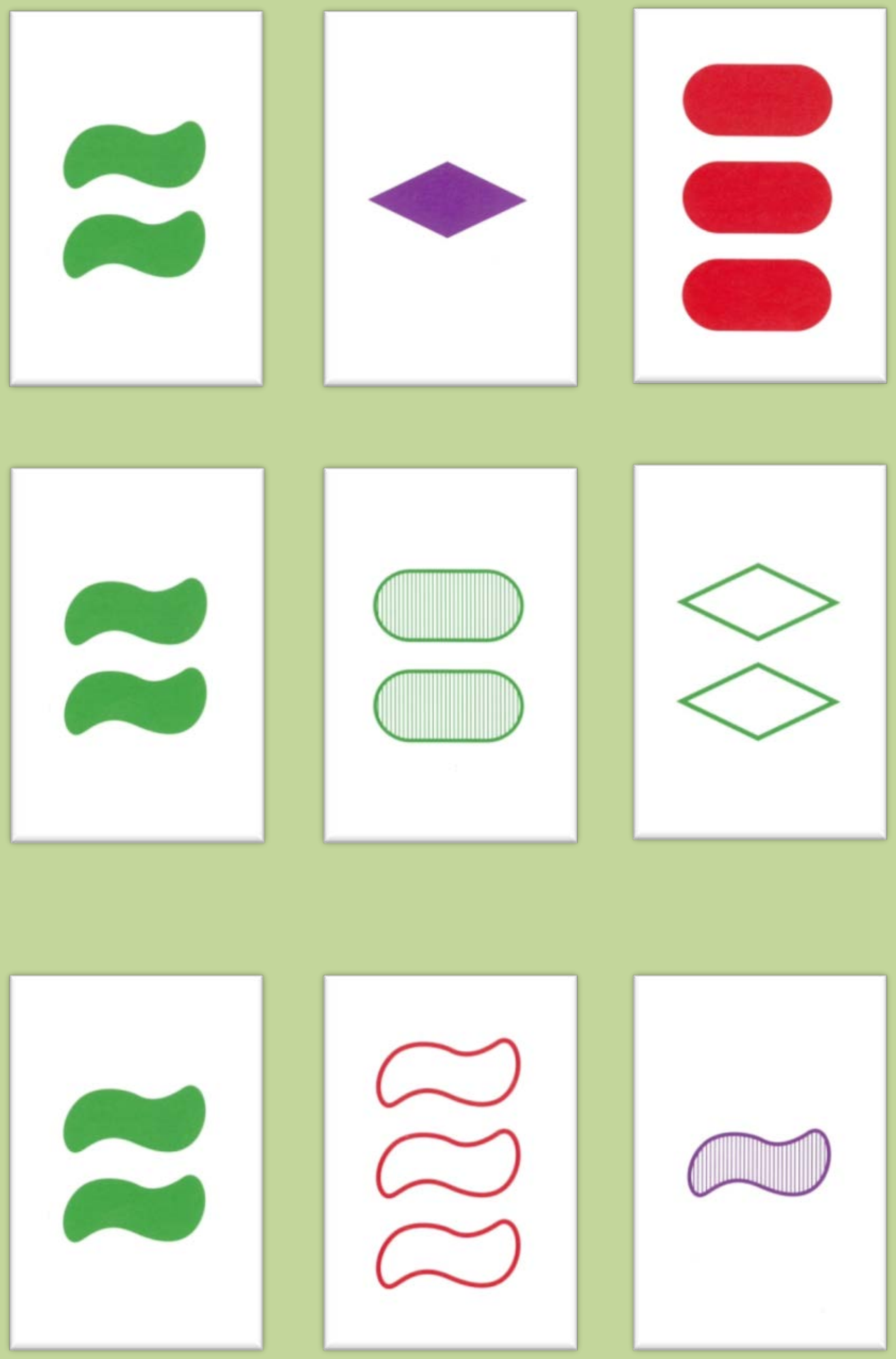


ADD

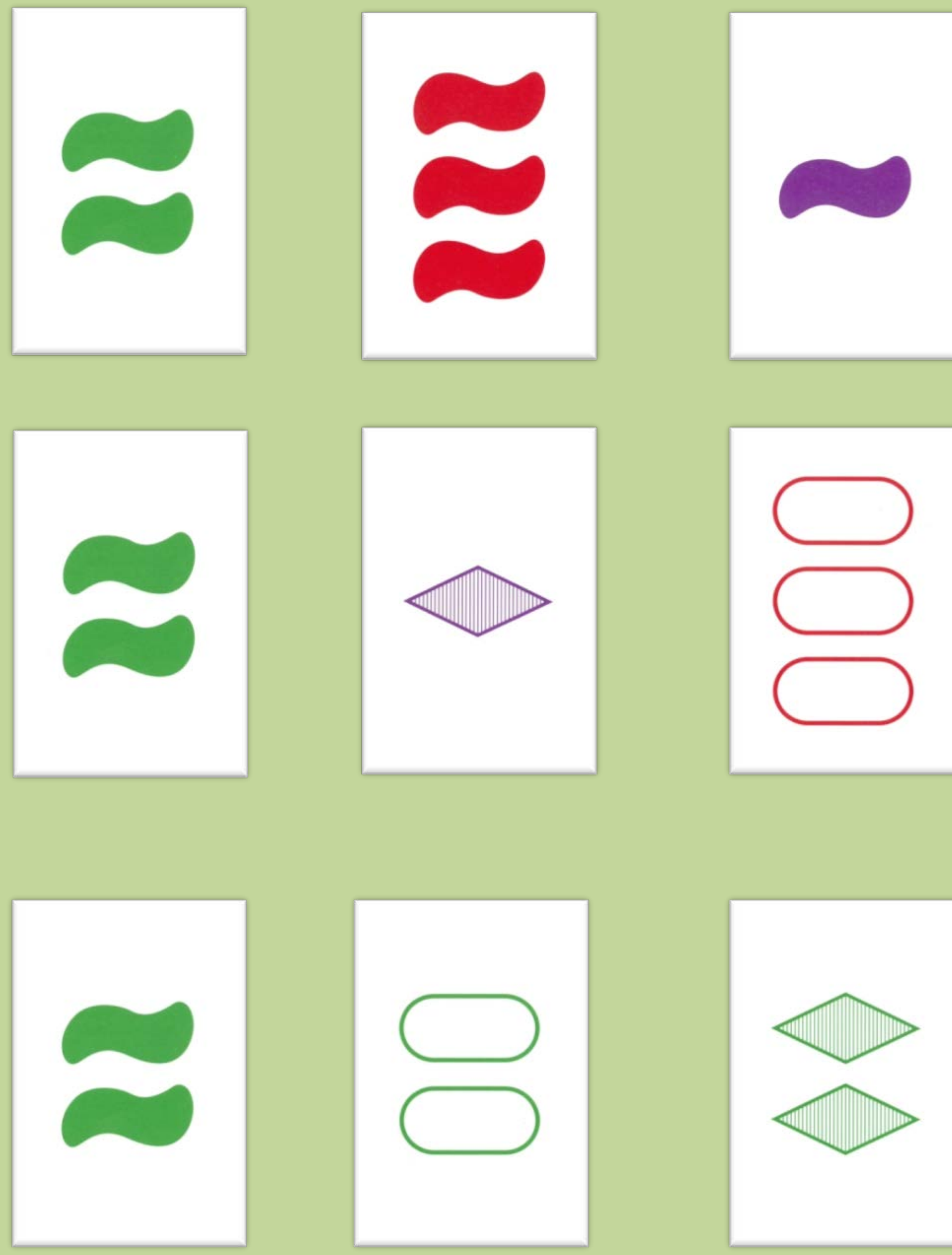




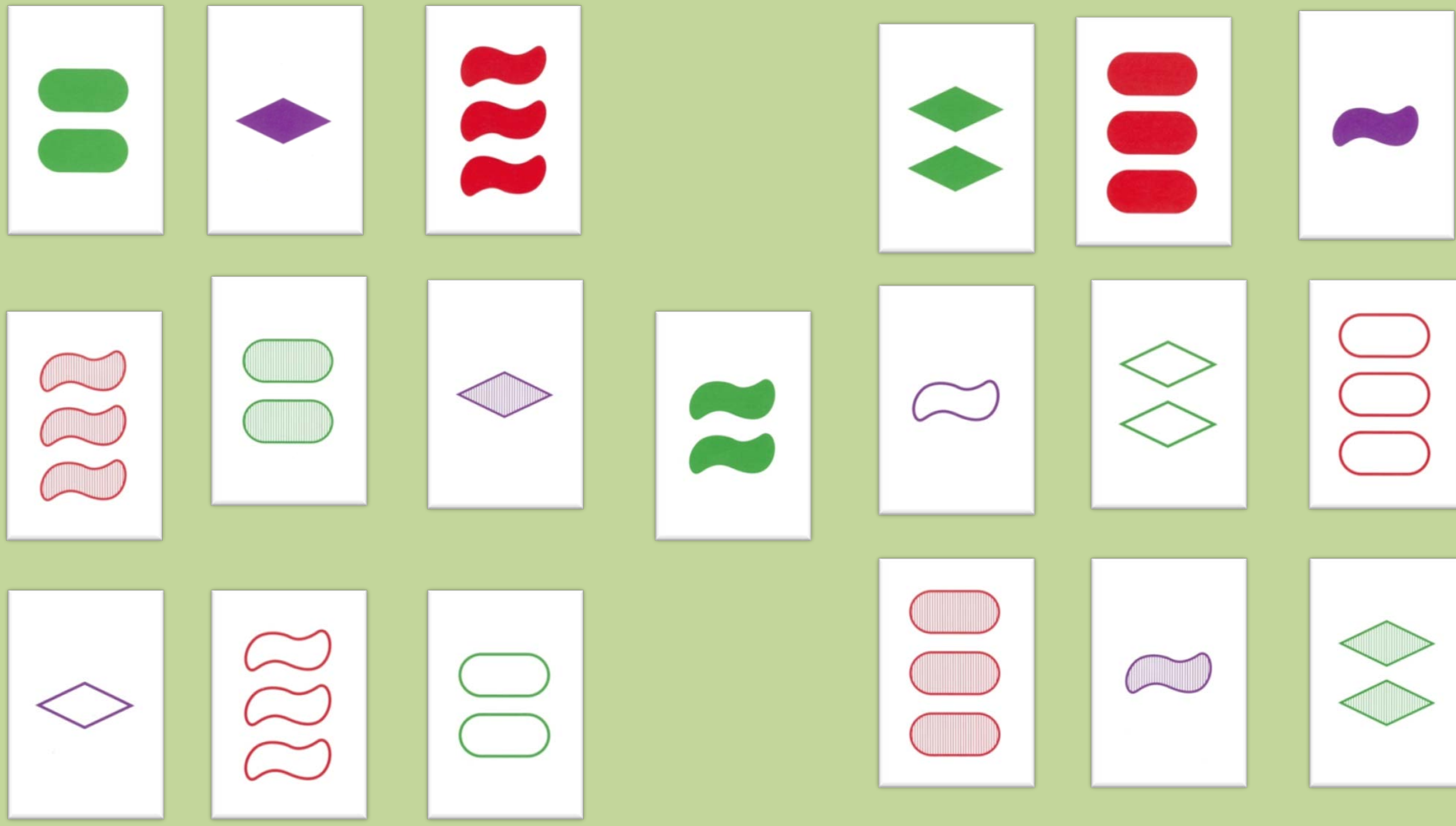
Pairing the new card with the first column will require three additional cards to complete each Set.



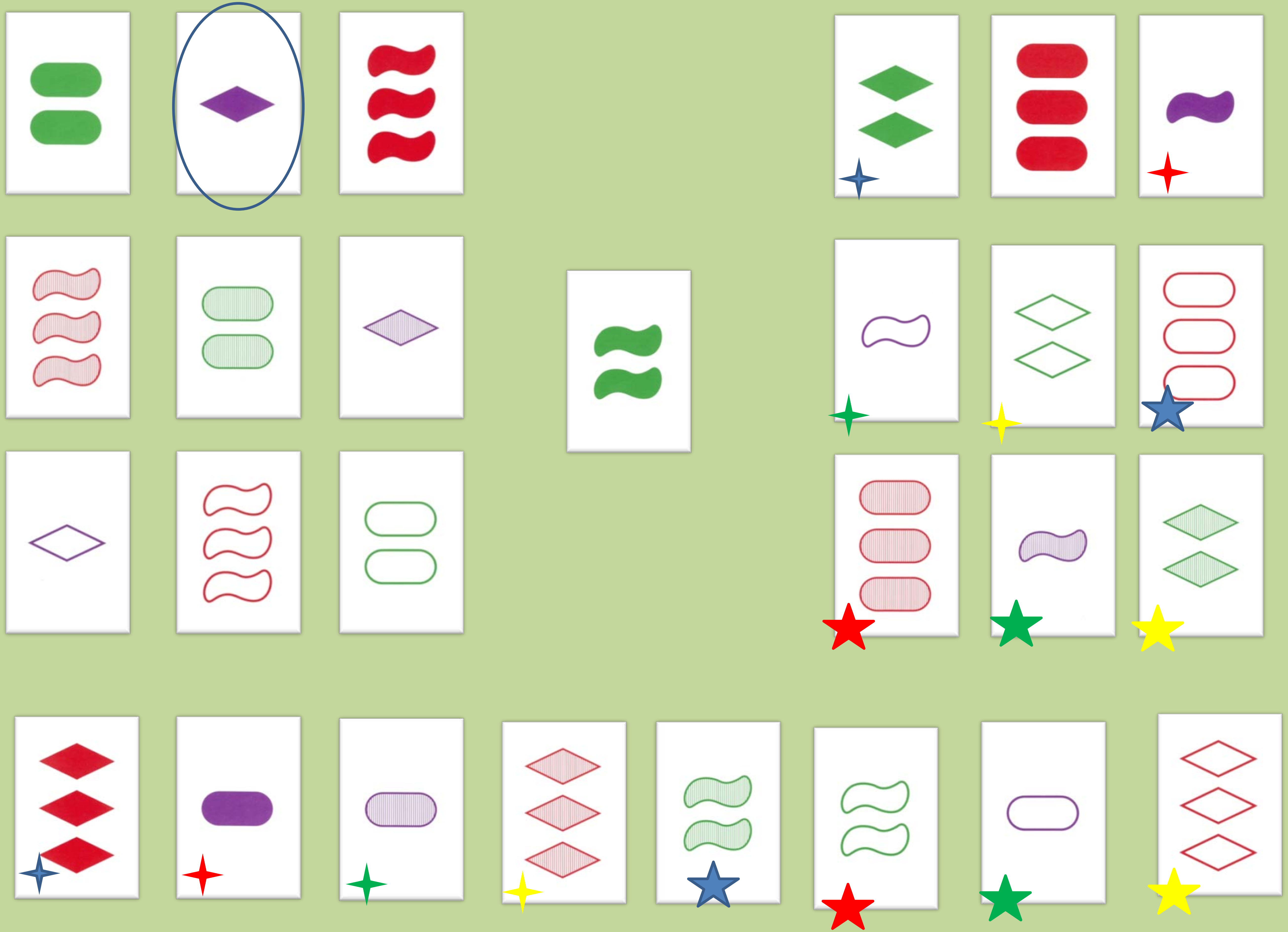
New card with second column

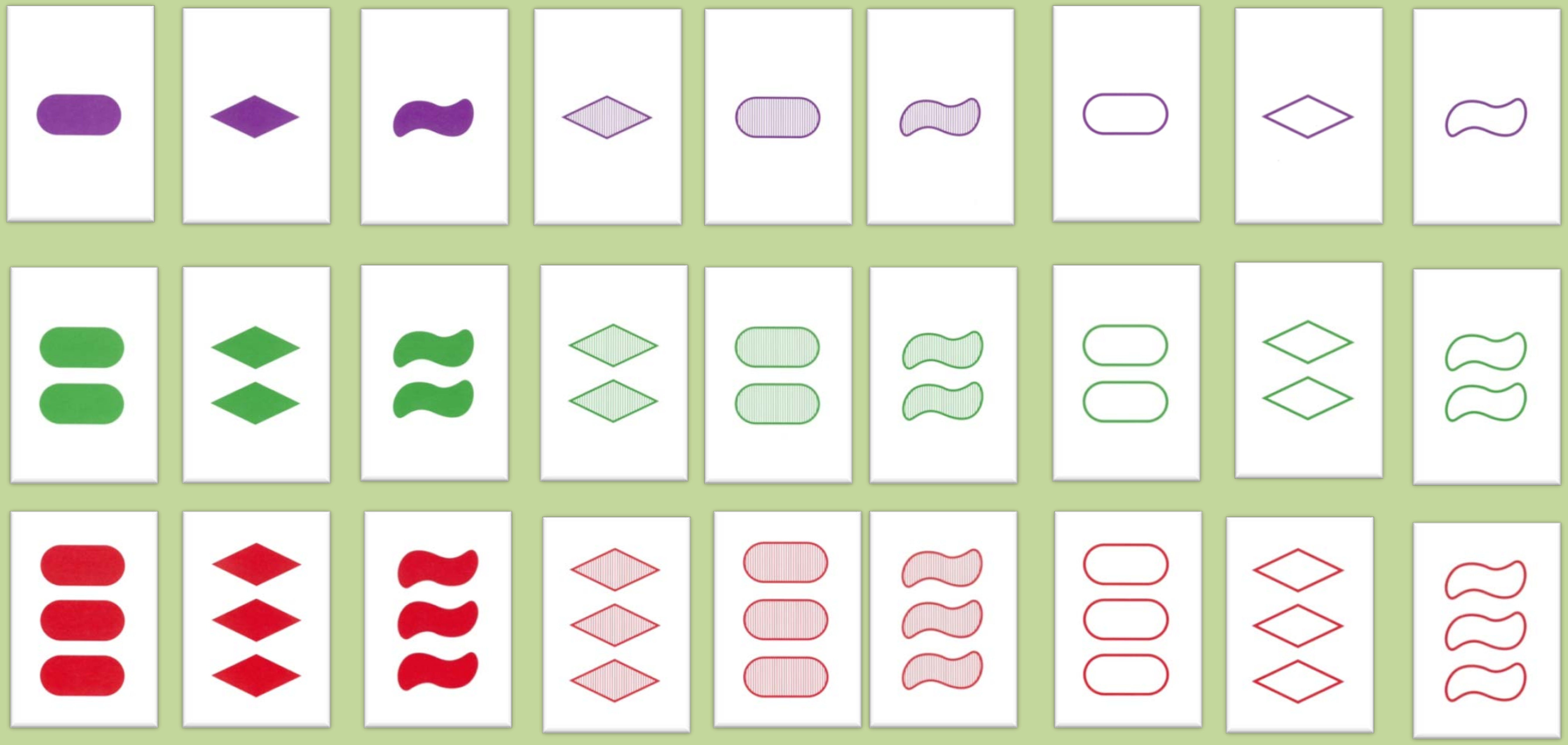


New card with third column



Do we have all possible Sets?

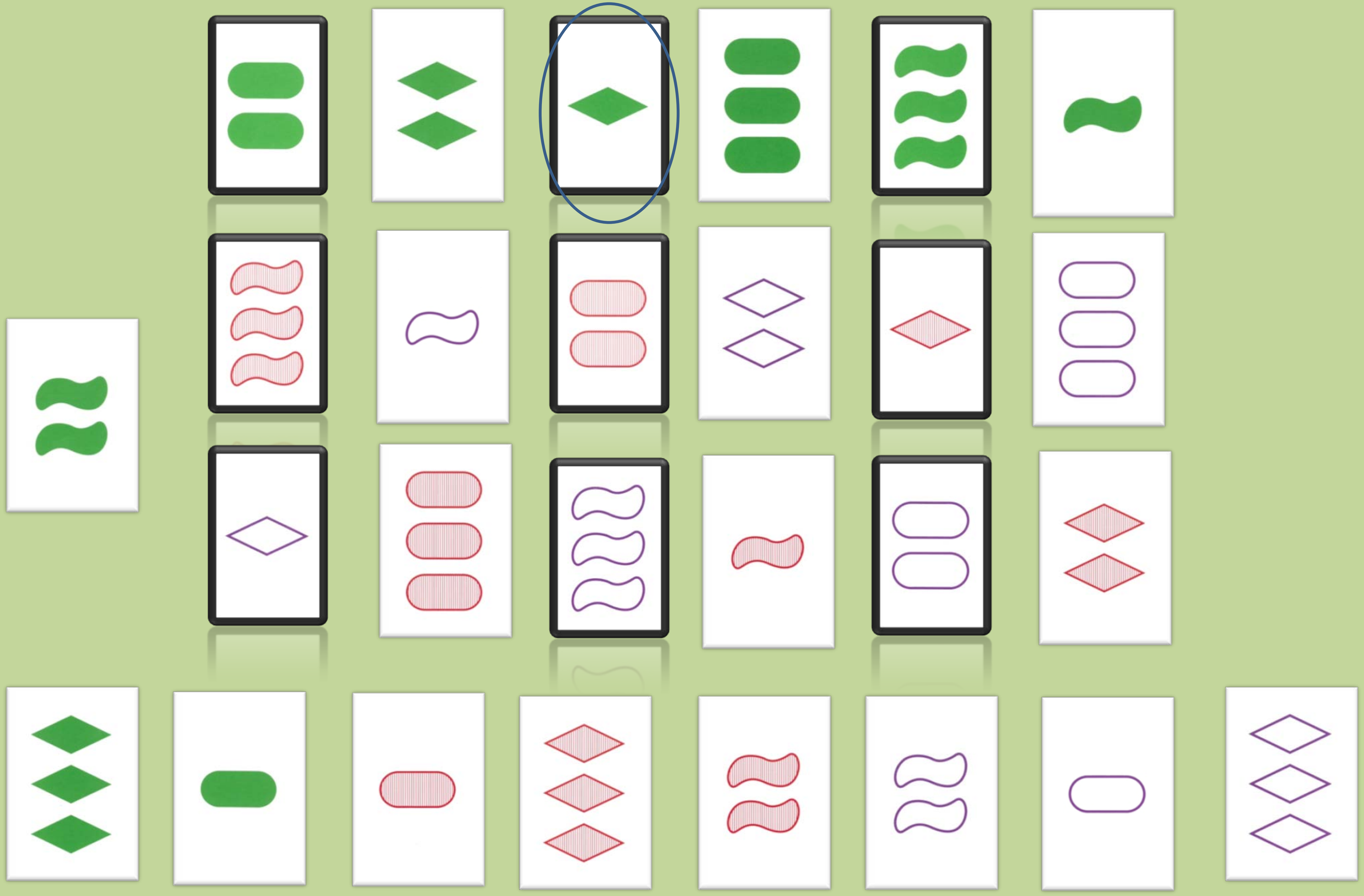


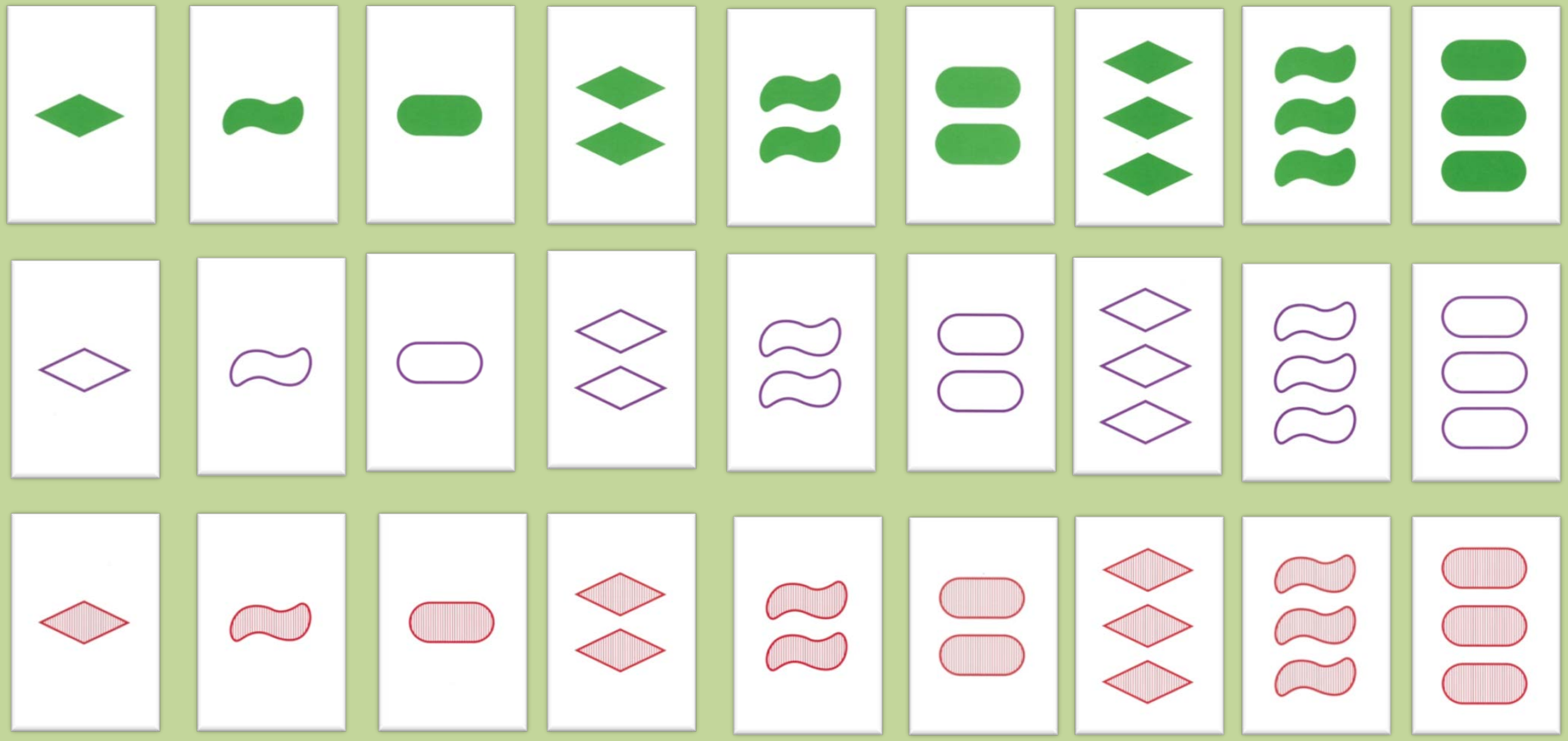


This plane consists of all one purple, all two green, and all three red.

We ended up with 27 points(cards) in our hyperplane.

Will this always happen?





This plane consists of all solid green,
all plain purple and all striped red.

How many distinct hyperplanes exist?

Can you classify the different types of hyperplanes as we did earlier with the Sets?

Coordinatization

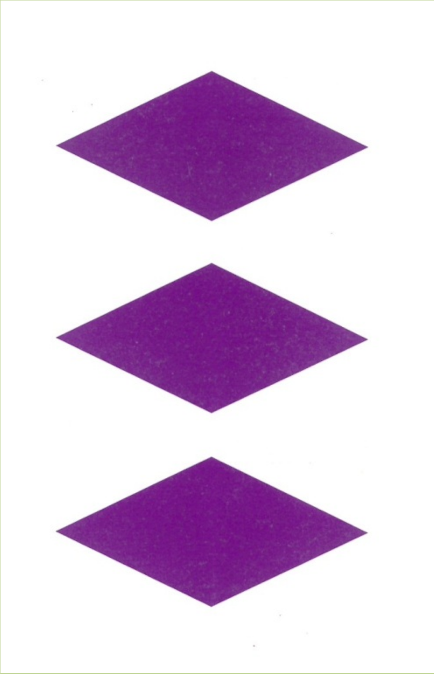
We can assign a four-tuple to each of the cards in the Set deck, using the values of 1, 2 or 3.

Values

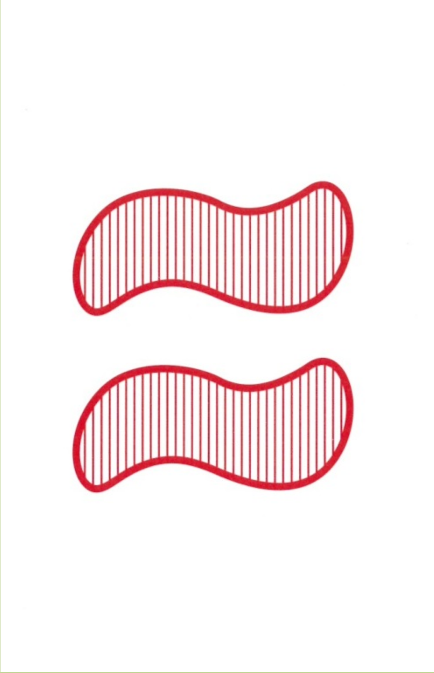
Coordinate	Number	Shading	Color	Shape
1	1	Plain	Red	Diamond
2	2	Striped	Green	Oval
3	3	Solid	Purple	Squiggle

The 2, striped, green, diamond card will be

(2, 2, 2, 1)



(3,3,3,1)



(2,2,1,3)

- To add or subtract in our system, we go coordinate by coordinate. For example, (2,1,1,2) + (1,1,2,1) is (3, 2, 3, 3)

- However, (2,3,1,2) + (3,1,2,1) is (5,4,3,3) and we have values that are not part of our original value set.

- To keep the results in our original set, we can add modulo (“mod’) three. This means we only record the remainder when the result is divided by three.

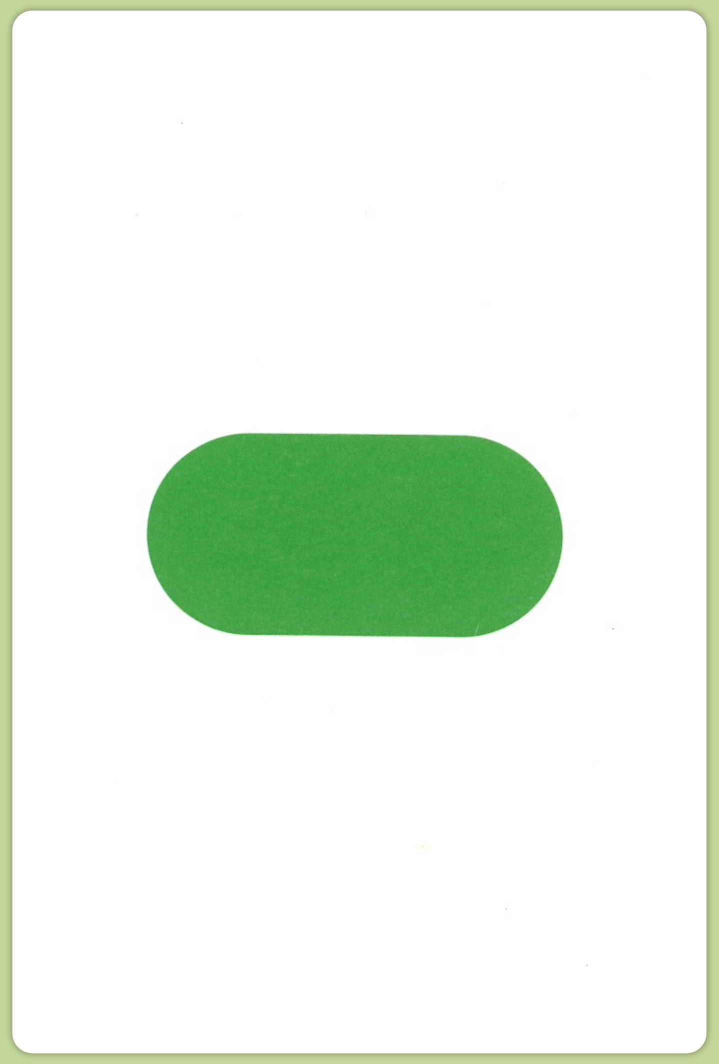
- Thus, (3,2,3,3) becomes (0,2,0,0) and (5,4,3,3) becomes (2,1,0,0)

An addition table modulo three would be:

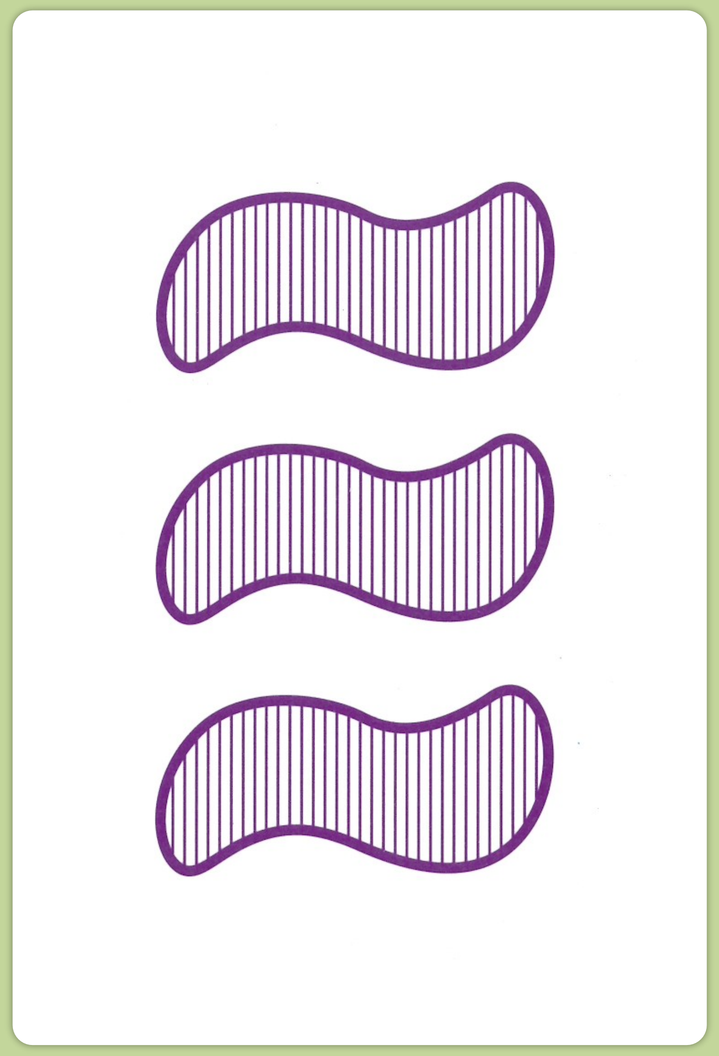
+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Our new assignment of values

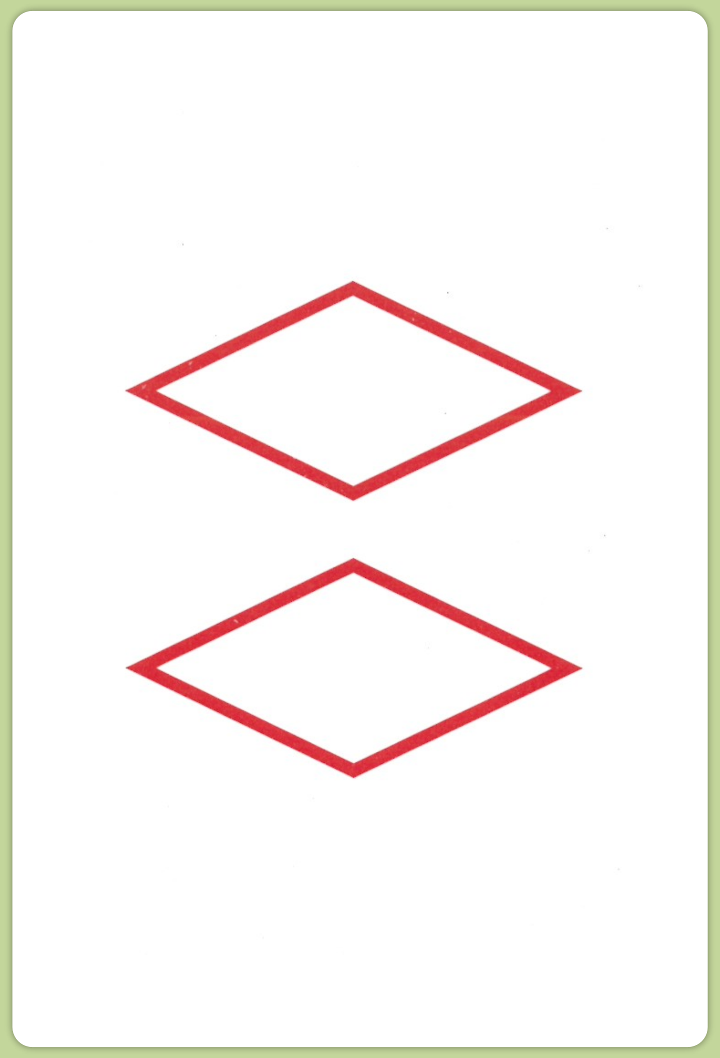
Coordinate	Number	Shading	Color	Shape
1	1	Plain	Red	Diamond
2	2	Striped	Green	Oval
0	0	Solid	Purple	Squiggle



(1,0,2,2)



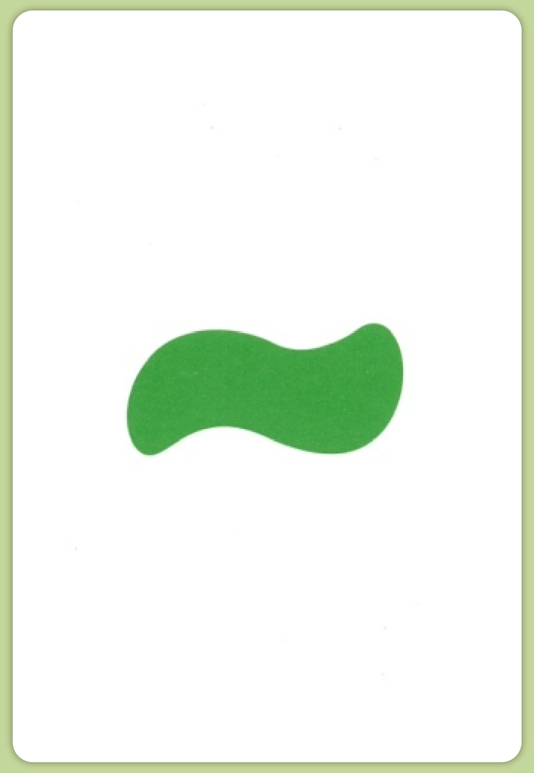
(3,2,0,0)

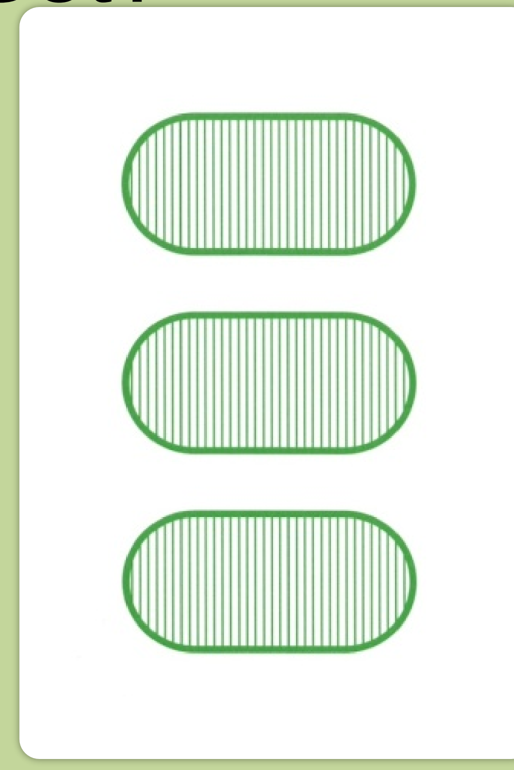


(2,1,1,1)

(0,0,0,0)

What happens when you add the coordinates of three cards that form a Set?





(1,0,2,0) (2,1,2,1) (0,2,2,2)

(0,0,0,0)

- The coordinates of the cards that form a Set will always have a sum mod 3 of (0,0,0,0).

- It can be easily shown that three cards form a Set if and only if their sum mod 3 is (0,0,0,0).

- If we represent the cards as A, B, and C, then three cards A, B, and C form a Set if and only if $A + B + C = 0 \bmod 3$.

Questions

- How many lines pass through a given point in the Set universe?
- How many planes pass through a given point?
- How many planes pass through a given line?
- How many different lines are there parallel to a given line?